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Introduction

While the Pareto law[1] is a useful first approximation to many size distributions in economics, one frequently encounters in plots of these distributions on double log paper a curve that is concave downward in shape instead of the straight line predicted by the law.[2] One possible source of this curvature is a violation of the first of the assumptions that Simon (1955) uses in his model to generate the Pareto law—namely, the assumption that percentage rates of growth are independent of size, otherwise known as Gibrat's law of proportionate effect. If, in fact, growth rates are negatively correlated with size, then there will be more cities or firms in the intermediate size classes than would be anticipated from the Pareto law, and an abundance of middle-sized firms or cities is precisely the meaning of the curvature. Supporting evidence for this hypothesis is the fact that for populations whose size distributions have this curvature, the empirical literature does tend to report an accompanying inverse relationship between growth rates and size.[3]

I am grateful to Yuji Ijiri and Herbert Simon for valuable comments on this and a previous draft of this paper and to an anonymous referee for extensive comments and suggestions, many of which have been incorporated into the draft here published.

[1] The Pareto law states that "the logarithm of the precentage of units with a size exceeding some value is a negatively sloped linear function of the logarithm of that value" (Klein 1962, p. 150). Throughout this paper, I shall use the word distribution to mean the greater-than cumulative, as is customary in the literature. The rank-size relationship or distribution is another name for the greater-than cumulative and will be used here interchangeably.

[2] Instances of this curvature abound in the empirical literature on firm, income, and city size distributions. Among the earliest to note it were Shirras (1935, p. 669) and Champernowne (1953, p. 347) for the income distributions of India and England. For similar evidence on city and firm size distributions, see, among others, Steindl (1965, p. 194) and Parr and Suzuki (1973, p. 346). My dissertation contains a more extensive list of such references.

[3] Negative correlations between growth rates and sizes of metropolises have been reported by a number of researchers in recent years. See, for example, Berry (1973, p. 104) and Leven (1973, p. 355). For similar evidence concerning firms, see Steindl (1965, pp. 214–15) and Jacquemin and Cardon de Lichtbuer (1973, pp. 402–3); for personal incomes, see Thatcher (1971). My dissertation contains a more extensive list of such references.

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One interpretation of this latter phenomenon in the case of firms, and the natural one to economists, perhaps, is that the increasing size of a firm acts in some manner to inhibit its rate of growth—that there is something inherent in the very nature of size that causes a progressive decline in this rate as it expands its activities. The conventional description of this tendency among the firms of an economy is that they are subject to decreasing returns to scale. By this interpretation, therefore, the presence of curvature in plots of firm size distributions on double log paper for an economy is nothing more than a sign of diseconomies of size in that economy.

In a recent paper, however, Ijiri and Simon (1974) indirectly argue against this interpretation by demonstrating that curvature may arise in the absence of any explicit violation of Gibrat's law, an assumption that is frequently taken in the economics literature to be equivalent to that of constant rather than decreasing returns to scale. According to Ijiri and Simon, curvature may result instead simply from the existence of a strong autocorrelation in growth rates over time, a fact about the growth of firms which was not allowed for in the growth process Simon (1955) originally postulated to account for the Pareto law.[4] In using this fact about the growth of firms to explain the observed curvature, however (or so Ijiri and Simon seem to argue), no recourse is needed to an additional assumption of a negative correlation between growth rates and size.

What I shall show here is that it is not autocorrelated growth per se but rather other features of the model in which this autocorrelation is embedded that generate the curvature. Specifically, in their model autocorrelated growth is modeled in such a way that growth dies out with the increasing age of a firm (Ijiri and Simon 1964, pp. 86–87; 1974, p. 322).[5] If autocorrelated growth rates are not accompanied by this inverse relationship between growth rates and age but rather are modeled so that "leads and lags dissolve with time," to use the language of Nelson

[4] The data reveal a strong element of persistence in most economic processes. For evidence on metropolitan growth rates, see Bogue (1953, p. 35) and on growth rates of firms, Ijiri and Simon (1967, p. 354) and Singh and Whittington (1975, pp. 21–22). Persistency in the growth rates of urban areas appears to be significantly higher than that in firm growth rates (see n. 10 below). Personal income and wealth data have been insufficiently detailed to date to determine if their year-to-year rates of change are likewise highly correlated. In his revision of his master's thesis, Champernowne (1973, p. 204) conjectures that the correlation between logged increments of income this year and last, for a given individual, will lie somewhere between .2 and .8. An indirect estimate of the correlation coefficient by Creedy (1974, p. 411) suggests, however, that it is very small.

[5] And, therefore, since age is positively correlated with size, there will be a negative correlation between size and growth rate for any cross-sectional sample of firms. Though ceteris paribus growth rates are independent of size—"the ceteris paribus assumption being that the previous growth of the firms being compared took place at about the same time" (Ijiri and Simon 1964, p. 81)—the likelihood of ceteris paribus ever holding for two different-sized firms declines with the absolute difference in their sizes.

and Winter (1973, p. 448), then curvature of the type observed will not arise.[6] This is demonstrated via the continuous model of autocorrelated growth presented in Ijiri and Simon (1967), which is really nothing more than Gibrat's model (1931) with nonindependent growth ratios. I conclude that autocorrelated growth by itself cannot be the source of the observed curvature, and that therefore the latter may very well be a reliable sign of diseconomies of size in an economy (or of some characteristic closely associated with size, such as age).

A Model of Autocorrelated Growth in which Firm Sizes Are Continuous Random Variables

In the literature on the size distribution of firms, there have evolved essentially two representations of the growth processes underlying these distributions. The first follows upon a model of Simon (1955) originally developed to account for certain statistical regularities observed in the distribution of word frequencies. It is this model that Ijiri and Simon have built upon in their subsequent papers on the subject of firm size distributions (1964, 1971, 1974). (An exception is their 1967 paper, which really belongs to an entirely different tradition of model, to be described below.) The more elaborate models presented in Steindl (1965) may also be fairly viewed as having the basic structure of this model at their heart.

It is a model in which (*a*) the sizes of firms are integer valued, and (*b*) the smallest size is normalized to one. The most straightforward revision of this model to allow for autocorrelation in the growth process is that presented in Ijiri and Simon (1964, 1974). Unfortunately, the model so revised has the added feature that growth rates decline with age of firm. It is therefore quite conceivable that it is this latter feature rather than autocorrelation per se which leads to curvature in the firm size distribution. To determine whether this is so, it is necessary to model autocorrelation in such a way that this negative correlation between age of firm and growth rate is absent. I have found no very appealing way of doing this within the discrete structure of the Ijiri-Simon model.

Fortunately, there is a different tradition of model which treats the size variable as a continuous one and is probably best exemplified in the paper by Hart and Prais (1956), which followed the work of Gibrat (1931). Unfortunately, the Gibrat model is in several respects distinctly more limited than that of Ijiri and Simon.

First, as Champernowne (1956) and Kendall (1956) note in their discussions of the Hart-Prais paper, the Gibrat model does not allow for firm entry at the smallest size (in contrast to that of Ijiri and Simon),

[6] This is in conflict with the results reported by Nelson and Winter, however. See Nelson and Winter (1973, p. 447).

possibly because it is much more cumbersome to have this feature in a continuous model. Second, it does not allow for autocorrelation in the growth process; the latest paper by Ijiri and Simon (1974) demonstrates that their model, again by contrast, is able to do so with only a minor loss of analytic tractability. One is therefore unable, by a straightforward application of the Gibrat model, to determine the extent to which the discrete model of Ijiri and Simon misrepresents the effects of autocorrelation.

To do so would seem to require a synthesis of the two models, which is what I shall attempt in this section. Since it is clearly desirable to treat the sizes of firms as continuous variables, in this respect I shall follow the model of Hart and Prais. On the other hand, since entries of new firms do take place at a fairly regular rate and since growth rates are serially correlated over time, I will allow firms to enter the existent population at a minimum size and then proceed to grow in accordance with a series of nonindependent draws from a growth-rate distribution, as in the Ijiri-Simon model. In contrast to the Ijiri-Simon model, however, the draws are made blind to age as well as size, so that the expected growth rate of any given firm over its future is independent of its current state. At the same time, this does not preclude the possibility, which seems to be implicit in Ijiri and Simon's exposition, that momentarily slow-growing firms will congregate in certain size classes and thereby give the impression that these sizes are inimical to growth.

Models of this form have so far resisted my attempts at analytical solution, and I have therefore sought the properties of the size distribution that they generate via simulation. The remainder of this section describes one specific model incorporating the features described above and the next the results of a simulation experiment performed on it.

The basic structure of the autocorrelation component of this model, interestingly enough, has already been worked out by Ijiri and Simon in their 1967 paper, although the approach was not pushed toward an explicit treatment of the steady-state distribution of firms implied by it.[7] In what follows I have borrowed heavily from that paper, in some places using their own language. I have elected not to disentangle what is theirs from what is mine, in the interest of readability.

Let S_{it} be the size of the ith firm at time t and r_{it} the growth ratio that relates S_{it} to $S_{i(t-1)}$, so that

$$S_{it} = r_{it}S_{i(t-1)}. \tag{1}$$

Let us further decompose r_{it} into two factors: one is a growth factor

[7] In Ijiri and Simon (1974, p. 322), the empirical results of the 1967 paper are presented as confirming the assumptions of the model presented in the 1964 and 1974 papers. For those familiar with these papers, the purpose of the present paper may be thought of as an exegetic one in which the two models advanced in these papers are shown to be of two quite different growth processes.

applicable to the ith firm only (the individual growth factor), ρ_{it}, and the other a growth factor that affects equally all firms in the economy, $\bar{\rho}$. Thus,

$$r_{it} = \rho_{it}\bar{\rho}. \quad (2)$$

Hence,

$$S_{it} = \rho_{it}\bar{\rho}S_{i(t-1)}. \quad (3)$$

Assume that the individual growth ratio, ρ_{it}, in the tth period is the product of the growth ratio of the same firm in the $(t - 1)$ period and a random factor, ε_{it}, which is distributed independently and identically for every firm and for every t—that is,

$$\rho_{it} = \varepsilon_{it}\rho^{\beta}_{i(t-1)}, \quad (4)$$

where β is some constant in the range $0 \leqq \beta < 1$, and ε_{it} has mean 1 and variance σ^2.

If $\beta = 0$, then we have the familiar Gibrat model that Hart and Prais have employed in their work. On the other hand, even if $\beta > 0$ (growth rates are positively correlated over time), (4) together with (3) is still consistent with Gibrat's law since the expected growth ratio over the future is independent of current firm size and age. That is, since $\beta < 1$, growth rates will have a tendency to regress toward the average growth rate of the entire economy, $\bar{\rho}$.

To complete the description of the model, there remains only the question of entry, ignored in the Hart and Prais model as well as in the model of Ijiri and Simon (1967) just presented. In the latter's discrete model, however, firms enter at the minimum size, 1, at some constant rate, α. This rate is measured empirically (see Simon and Bonini 1958, p. 611; Engwall 1968, pp. 142–43; Vining 1974, p. 327) as the share of total growth for the firms above a given minimum size over a given time period which is due to new firms entering this population during this time period. By the notation introduced above, the total growth due to old firms, OG_t, between the beginning and the end of time t is given by

$$OG_t = \sum_{i=1}^{k(t-1)} [S_{it} - S_{i(t-1)}], \quad (5)$$

where $k(t - 1)$ is the number of firms in the economy at the end of time $(t - 1)$. If the share of total growth by new firms is α, then the total number of new assets to be added to the economy via new firms should be, on the average,

$$NG_t = OG_t[\alpha/(1 - \alpha)]. \quad (6)$$

New firms are entered at the end of time t. The number of such firms is given by

$$NC_t = NG_t/S_{\min}, \quad (7)$$

where $S_{\min}$ is the minimum size. Since NC_t must be an integer, in the actual simulation of this process, it is rounded off to the nearest integer and the remainder (negative if rounded up and positive if rounded down) is carried over to the next time period and added to NC_{t+1}. Thus, in the actual simulation, NC_t is calculated exactly by

$$NC_t = INT[(NG_t/S_{\min}) + .5 + (R_{t-1}/S_{\min})], \quad (8)$$

where, $R_{t-1} = NG_{t-1} - (NC_{t-1}S_{\min})$, R_0 equals zero, and INT is the Fortran function that converts the number in its argument consisting of a whole number plus a fraction to that whole number. For the next time period $t + 1$, these new firms are regarded as old firms with size $S_{\min}$ and growth ratios $\rho_{i(t+1)}$, calculated from (4) with ρ_{it} equal to one.

The simulation proceeds, therefore, in two steps. At the first, each existent firm's growth ratio, ρ_{it}, is computed via (4); β is held constant throughout the simulation, and ε_{it} is drawn from a lognormal distribution with mean 1 and variance σ^2.[8] Each firm is then "grown" to its next period's size and the total increment of assets for the old firms, OG_t, is computed via (5). At the second step, the number of new firms to be added is computed via (6) and (8); their initial growth ratios are assumed to be one in the calculation of their growth rates in the succeeding time period, while their initial sizes are taken to be the minimum firm size, $S_{\min}$. The simulation executes these two steps for a large number of iterations. At the end, the existent firms are arrayed in order of decreasing size and plotted against their ranks on double log paper. The next section of this paper presents these plots for different values of β with σ, $\bar{\rho}$, and α held constant.

Rank-Size Relationships Generated by the Model

This section compares the rank-size relationships generated by simulation runs of the model described in the last section for different degrees of autocorrelation, that is, for different values of β. All other parameters of the model ($\bar{\rho}$, α, and σ) were held constant during these runs. Specifically, $\bar{\rho}$ was set equal to 1.0285, σ to 0.0095, and α to 0.05. The particular values of the first two were chosen simply to insure that firms rarely

[8] This implies lognormally distributed growth rates, i.e., a distribution of growth rates skewed to the right. The empirical literature tends to substantiate this assumption. See Madden (1956, p. 250) for the evidence concerning cities and Hart and Prais (1956, pp. 170–71) for that concerning business firms. Mandelbrot (1963, pp. 30–31), however, argues that the distribution of logged growth ratios is closer to a two-tailed Pareto than the normal distribution that one would expect from lognormally distributed growth rates —i.e., that there are many more extreme observations than one would expect from a normal distribution.

decline in size: it will be remembered that the Ijiri-Simon model is one of a pure growth process, and therefore in order to make a valid comparison with that model the one presented here must also possess that property. Although the value for α is somewhat lower than that reported in the literature,[9] higher values were found to demand too much computing time for the number of time periods that I wanted to run the simulation. Run time is a direct function of the number of firms in the population, which are in turn growing at the (exponential) rate of $(\bar{\rho} - 1)/(1 - \alpha)$. With $\bar{\rho} = 1.0285$ and $\alpha = 0.05$, the aggregate rate of growth is 3 percent per time period.

It is customary in simulation studies to test the procedure on a special case of the model for which there is a known analytic solution. In the present case, such a case occurs when $\beta = 0.0$. That is, one would anticipate from the analytic results presented in Simon (1955) for a nonautocorrelated growth process that the steady-state relationship between rank and size would trace out an approximately straight line on double log paper with an absolute slope of $(1 - \alpha)$, or .95 if $\alpha = 0.05$, for $\beta = 0.0$. The first graph of figure 1 (for $\beta = 0.0$) bears out this expectation. The rank-size relationship, after 300 years of simulated time, closely conforms to the predictions of the Simon model: it is linear on double log paper with absolute slope, $1 - \alpha$, or .95. (There are divergences from linearity in the upper end, which are due to the fact that the slope converges only asymptotically to $[1 - \alpha]$ from above [see Vining 1975]. This means a somewhat larger slope than $[1 - \alpha]$ initially; therefore, as observed in fig. 1, the points will lie above the predicted rank-size relationship for large S and small R.) On the basis of this result, I conclude that the simulation procedure is an accurate embodiment of that model. However, the reader is invited to inspect the simple Fortran program used to implement the procedure, available from me on request.

To compare the rank-size relationships for different degrees of autocorrelation in the growth process, I ran the model for four different values of β: 0.3, 0.5, 0.7, and 0.9.[10] Typical log-log charts of the rank-size relationships generated by the model for each of these four values of β, again after 300 years of simulated time, are presented in the remaining graphs of figure 1. While there is evidence, in agreement with the Ijiri-Simon results (1974), that the degree of curvature increases with the degree of autocorrelation—that is, with higher β—the curvature is convex rather than concave and lies on the down side of the Pareto curve rather

[9] See, for firms, Simon and Bonini (1958, p. 612) and Engwall (1968, p. 153); for cities, see Vining (1974, p. 327).

[10] For two 4-year periods, Ijiri and Simon (1967, p. 354) estimate β to be around one-third for the largest 100 corporations in the United States, while I have estimated β for American urban areas, for two 10-year periods, to be around .75 (see Vining [1975] for a

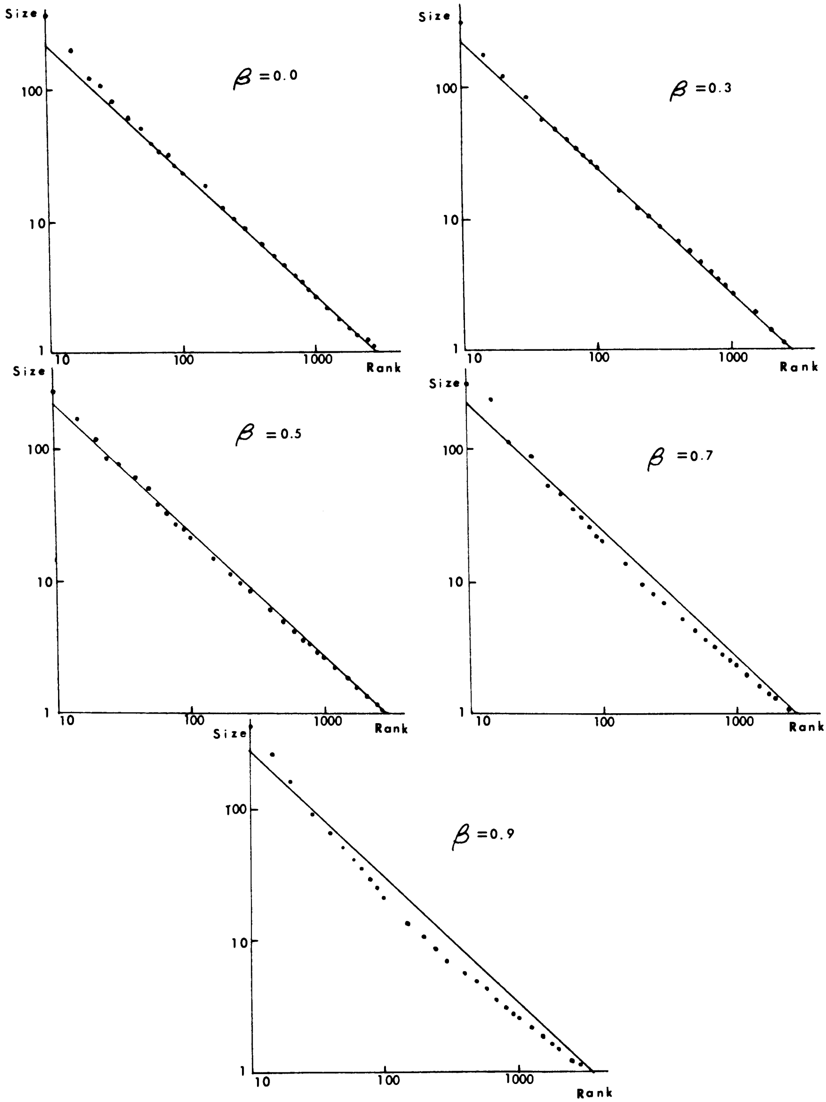

FIG. 1.—Typical rank-size relationships generated by the simulation for β = 0.0, 0.3, 0.5, 0.7, and 0.9 (The solid line is the rank-size relationship predicted by the Simon model [1955] and is drawn with absolute slope .95 from the smallest size.)

than the upper as reported by Ijiri and Simon. Convex curvature generally reflects an underlying positive correlation between growth rates and size: the larger number of firms generated when $\beta = .9$ is also a sign of a positive correlation between growth rates and size.[11]

It seems clear, therefore, that autocorrelation by itself (i.e., with the expected long-run growth rate for a given firm independent of its current age and size) cannot be the source of the curvature actually observed in empirical rank-size relationships in that it cannot be the source of the negative correlation between growth rates and size which is the immediate cause of this curvature. Quite the opposite, autocorrelation appears to generate a positive correlation between size and growth rate, at least for a high degree of autocorrelation in the growth process, and therefore a higher degree of concentration than would occur if growth rates were independent over time.[12] I conclude that for autocorrelated growth rates to be compatible with the observed curvature, the growth rates for a given firm must in addition have expected values that are a declining function of that firm's size or of some variable highly correlated with size, such as age. Of course, to constrain growth rates in this fashion is to rob the original hypothesis (i.e., that autocorrelation in the absence of

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description of the data and fitting procedures). In order to compare the magnitudes of these two estimates of β, it is necessary to standardize them for the same time period. After adjusting both estimates to an annual basis (by a method easily adopted from Telser [1967]; see Vining [1975]), β turns out to be .65 and .95 for firms and urban areas, respectively. Thus, if one were to regard the year as the "natural" time period for most economic processes, β for two of these processes appears to be well over one-half. These estimates, of course, are intended only to give the reader some rough idea of what degree of autocorrelation there is in two important processes in economics, city and corporate growth, so that he can look at the charts in figure 1 more intelligently.

[11] They are typical in the following sense: for all simulations, convexity in the rank-size charts appears with $\beta = .5$ and increases thereafter. The simulations for which the charts represent a typical outcome were replicated three times (i.e., with three different random number seeds) for three different initial size distributions (one of 10 firms with sizes arrayed in rank-size order with slope parameter equal to .95; one of three firms, again in rank-size order; and one of two firms, of sizes five and three, respectively).

[12] It is interesting to note that Steindl (1965, pp. 213–14) has argued on intuitive grounds that persistency in the growth process will generate this positive correlation and therefore a higher degree of concentration than would be normal if growth rates were independently distributed over time. Another interpretation of the increased convexity with increasing β has been offered by the referee: "As $\beta \rightarrow 1$, the AR(1) process, (4), approaches nonstationarity. In fact, it approaches a random walk. Thus, for large β, we would expect the behavior of the series to be quite different from that for β near 0. In particular, convergence to a steady-state distribution may be considerably slower. It is quite conceivable that the increased convexity with increasing β is due to this. You have already noted that the slight convexity at the upper end for $\beta = 0$ is due to the convergence to the asymptotic relationship from above. I can find nothing in your figure which would be inconsistent with the hypothesis that the observed convexity represents a shifting of the effect already noted for $\beta = 0$. Convergence to a steady-state distribution can be extremely slow in such processes. If true, the slow convergence could be an essential and important aspect of the model."

any such constraint will generate a negative correlation coefficient computed on a cross-sectional sample of growth rates and firm sizes) of much of its intrinsic interest. Naturally, if growth rates are ever declining, as they are if they are negatively correlated with age, they must also be strongly autocorrelated. However, it is the former and not the latter feature of the model that is the source of the observed curvature.[13]

A striking implication of this finding is that the rank-size rule, or Pareto's law, must then exist, where it does indeed exist, in a kind of balance between autocorrelated growth rates and the inhibitory effects of age on growth rates: the latter pull the distribution into concavity, the former into convexity. To judge from the empirical city and firm size distributions, however, the age effect appears to have recently come to dominate the autocorrelation effect.

Conclusion and Summary

This paper had the single objective of demonstrating that autocorrelation in firm growth rates cannot by itself account for the negative correlation between growth rates and size frequently observed in cross-sectional samples of corporations (as well as cities) and therefore cannot in turn account for the concave curvature observed in the distribution of their sizes on double log paper which is an expression of this negative correlation. The method used to demonstrate this fact was to simulate the continuous model of Gibrat with the added features of autocorrelated growth rates and expanding firm population. The resultant distribution of firm sizes showed convexity rather than concavity, implying a positive correlation between firm size and growth rates. Thus, it appears that the negative correlation between firm size and growth rates must be built directly and explicitly into the model if curvature like that observed is to be generated. In short, it cannot arise as a by-product of some weaker set of assumptions concerning firm growth.

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[13] Table 1 of Bogue (1953, p. 13) clearly shows this effect of age on standard metropolitan area (SMA) growth rates. Cities entering the SMA population at each decade (i.e., surpassing 100,000 population) show a higher average growth rate than those which entered in a previous decade and a lower average growth rate than those which entered in a later decade. Williamson and Swanson (1966, p. 53), in a multiple-regression study of the effects of size and age on city growth rates in the nineteenth century in this country, report a negative effect of age and no effect of size. That is, for a given size, the older the city, the lower the growth rate, on the average, while, for a given age, size has no effect on a city's growth rate. Bock and Farkas (1973, pp. 24–34) present similar evidence for American corporations.

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