Power Laws and the Origins of Macroeconomic Flucatuations

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Abstract

If firm sizes have a small dispersion (finite variance of sizes), microeconomic shocks lead to negligible aggregate fluctuations. This has led economists to appeal to macroeconomic (sectoral or aggregate shocks) shocks to explain macroeconomic fluctuations. However, the empirical distribution of firms is fat-tailed: it is fat tailed with an exponent around 1 (the variance is infinite). As this paper shows, in such a world, micro fluctuations aggregate up to non-trivial macro fluctuations.

Incidently, the model predicts several other features that are borne out in the data: It predicts that large countries have smaller volatility than small countries with a power law relationship identical to that found for firms (countries with a GDP of S have a volatility proportional to $S^{-\alpha}$ with $\alpha \simeq .15$). It delivers the distribution of GDP fluctuations close to the one found empirically (a modified Lévy distribution), and the time-series properties (hump-shaped impulse response) of deviations from trend.

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1 Introduction

This paper will propose an answer to a precise puzzle and a simple origin for macroeconomic shocks.

The "scaling" puzzle has been found in a series of papers by a team of physicists and economists, Amaral et al. (1997,1998), Canning et al. (1998), Lee et al. (1998). In a nutshell, firms and countries have identical, non-trivial, scaling of growth rate. Amaral et al. study how the volatility of the growth rate of firms of size[1] S changes with S. To do this, one divides the firms in a number of bins of sizes S, calculate the standard deviation of the growth rate of their sales $\sigma(S)$, and plots $\ln\sigma(S)$ vs $\ln S$. One finds a rougly affine shape, displayed in Figure 1:

$$\ln\sigma^{\text{firms}}(S) = -\alpha\ln S + \beta.$$

Exponentiating gives:

$$\sigma^{\text{firms}}(S) \sim S^{-\alpha} \tag{1}$$

A firm of size S has volatility proportional to $S^{-\alpha}$ with $\alpha = 0.15$. This means that large firms have a smaller proportional standard deviation that small firms, but this diversification effect is less strong that would happen if a firm of size S was composed of independent units of size 1 (which would predict that volatility decreases in $S^{-.5}$ rather than in $S^{-\alpha} = S^{-.15}$).

Canning et al. (1999) do the same analyses for country growth rates, and find[2] that countries of GDP of size S have also a volatility of size $S^{-\beta}$, with $\beta = .15$. The two graphs are plotted in Figure 1. The puzzling fact is that $\alpha = \beta$.We gather this as:

$$\sigma^{\text{GDP}}(S) \sim S^{-\alpha} \tag{2}$$

The equality of the exponent in (1) and (2) could be a fluke. But we view is a tantalizing fact that could guide us to insights about the origins of macroeconomic fluctuations.

[1] The measure of size can be assets, or sales, or number of employees. Those three measure give similar results.

[2] Another way to see their result is to regress:

$$\begin{aligned}\ln\sigma_i &= -\alpha\ln Y_i + \beta\ln \text{GDP/Capita} + \gamma\text{Openness} \\ &\quad + \delta\text{Gvt share of GDP} + \text{constant}\end{aligned}$$

where σ_i is the standard deviation of $\ln Y_{it}/Y_{it-1}$ and Y_i the mean of the Y_{it}. We run this over the top 90% of the countries to avoid the tiniest countries, and find $\alpha = .15$ with a standard deviation of .015.

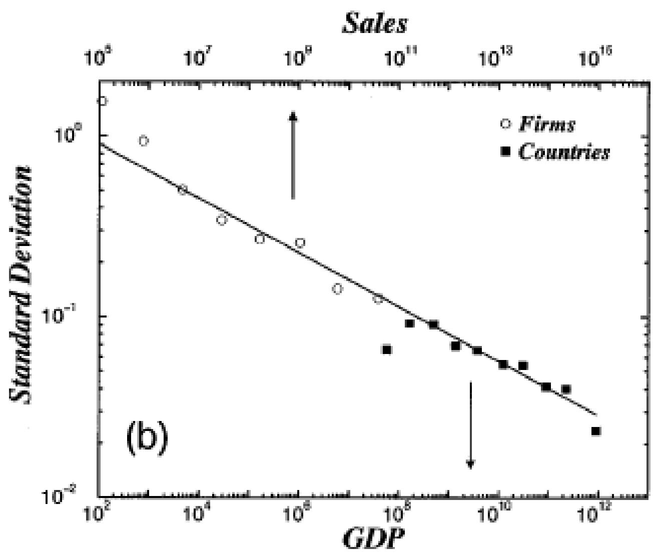

Figure 1: Standard deviation of the distribution of annual growth rates (log log axes). Note that $\sigma(S)$ decays with size S with the same exponent for both countries and firms, as $\sigma(S) \sim S^{-\alpha}$, with $\alpha = .15$. The size is measured in sales for the companies (top axis) and in GDP for the countries (bottom axis). The firm data are taken from the Compustat from 1974, the GDP data from Summers and Heston (1991). Source: Lee et al. (1998).

This paper will advance a simple reason for the identical scaling (1)-(2). Modern economies have many large firms. For instance, in the US, the sales of the top 20 firms account for about 20% of total US GDP. Suppose that there are idiosyncratic shocks to those firms. They will still add up to large GDP fluctuations, as those firms are large. Macroeconomic volatility will have as microeconomic volatility as its orgin. And the identical scaling (1)-(2) is then readily explained.

This puts more flesh on a "real business cycles" view of macroeconomic shocks. In a recent report of the McKinsey Institute (2001), 80% of the productivity growth in the service sector in 1997-2000 were due to one firm, Wal-Mart. Finland has a boom when one firm, Nokia, expands a lot. The Californian economy enters in the recession in the mid-90s after a few contracts are lost by firms in the defense industries, and a few big movies are disappointing. Hence real shock are not a common "productivity shocks", but well-defined shocks to individual firms[3].

We will present the argument with several degrees of sophistication. The simplest version of the argument will be developed in section 2. In section 3, we will propose a more in-depth treatment of the process. We will gain predictions about the shape of the fluctuations of the growth rate of firms and countries. We will argue that it is in fairly good agreement with reality. Section 4 will discuss the result, and its link with the literature.

2 The essence of the idea

2.1 Microscopic volatility

Imagine an economy with only idiosyncratic shocks to firms. We study its aggregate volatility, and call if the "microscopic volatility". Say that firm i produces S_{it}. In a year t, it has a change in size:

$$\Delta S_{it+1} = S_{i,t+1} - S_{it} = \sigma_i \cdot S_{it} \cdot \varepsilon_{it+1}$$

where ε_{it+1} are independent random variables with mean 0 and variance 1, and where σ_i is the volatility of firm i. For simplicity, we consider assume that all mean growth rates are 0. The total GDP is:

$$Y_t = \sum_{i=1}^{N} S_{it}$$

[3] They can propagate to the rest of the economy. There is a very large literature on those "propagation mechanisms". This papers focuses on the original shocks, not their propagation.

so that:

$$\frac{\Delta Y_{t+1}}{Y_t} = \frac{1}{Y_t}\sum_{i=1}^{N}\Delta S_{it+1} = \sum_{i=1}^{N}\sigma_i \cdot \frac{S_{it}}{Y_t}\cdot \varepsilon_{it+1}$$

so, given that the shocks are ε_{it+1} are uncorrelated:

$$var\frac{\Delta Y_{t+1}}{Y_t} = \sum_{i=1}^{N}\sigma_i^2 \cdot \left(\frac{S_{it}}{Y_t}\right)^2$$

We define $\sigma_{GDP}^{\text{Micro}}$ as the volatility of GDP fluctuations coming from the idiosyncratic micro shocks:

$$\sigma_{GDP}^{\text{Micro}} = \left(\sum_{i=1}^{N}\sigma_i^2 \cdot \left(\frac{S_{it}}{Y_t}\right)^2\right)^{1/2} \tag{3}$$

Hence the variance of GDP that arises from purely idiosyncratic shocks $\left(\sigma_{GDP}^{\text{Micro}}\right)^2$ of is the weighted sum of the variance of idiosyncratic shocks σ_i^2, with weights equal $\left(\frac{S_{it}}{Y_t}\right)^2$ to the squared share of output that firm i accounts for. We shall use equation (3) throughout the paper.

2.2 The $1/\sqrt{N}$ argument for the necessity of aggregate shocks

We first briefly recall the reason why, in macroeconomics, one usually appeals to common (or at least sector-wide) aggregate shocks. With a large number of firms N, one could expect the sum of their idiosyncratic shocks to be vanishingly small. Indeed, take firms of initially idential size $S_{i,t=0} = Y/N$, and identical standard deviation $\sigma_i = \sigma$. Then (3) gives:

$$\sigma_{GDP}^{\text{Micro}} = \frac{\sigma}{\sqrt{N}}.$$

This a version of the central limit theorem: the volatility of the average of N units is of order $1/\sqrt{N}$. To get an idea of the order of magnitude, suppose each firm has a production shock of $\sigma = 20\%$ per year, and take $N = 10^6$ firms. We get

$$\sigma_{GDP}^{\text{Micro}} = \frac{\sigma}{\sqrt{N}} = \frac{20\%}{10^3} = 0.020\%/\text{year}$$

which is just too small to account for the empirical size of macroeconomic fluctuations. This is why economists typically[4] appeal to aggregate shocks. But large firms, in modern economic, have a size much bigger than Y/N. We now explore the consequence of this.

2.3 Scaling and macroeconomic fluctuations when firms are non-atomistic

Modern economies have large firms. How big are those GDP fluctuations coming from their idiosyncratic shocks? We start with a rough order of magnitude. Suppose that those $J = 16$ top firms have idiosyncratic shocks with volatility $\sigma_i = \sigma = 20\%$. Say that each of them account for about $S_{it}/Yt = s = 1.5\%$ of GDP. Neglect the contribution of the other firms to GDP volatility. We get a total GDP volatility:

$$\begin{aligned}
\sigma_{GDP}^{\text{Micro}}(J) &= \left(\sum_{i=1}^{J} \sigma_i^2 \cdot \left(\frac{S_{it}}{Y_t}\right)^2\right)^{1/2} \\
&= J^{1/2}\sigma s \\
&= 16^{1/2} \cdot .015 \cdot .20 \\
&= 1.2\%.
\end{aligned}$$

This simple exercise gives just an order of magnitude. Doing the numerical exericise over the firms in Compustat[5], we get

$$\sigma_{GDP}^{\text{Micro}} = 1.27\%$$

This suggests that, quantitatively, our mechanism can account fairly large GDP fluctuations. In this thought experiment, shocks are iid. Macroeconomics has identified a variety of amplification mechanisms. Combining

[4] One route out of this has been taken by Jovanovic (1987), who observes that the multiplier is very large ($1/(1-\lambda) = M \sim \sqrt{N}$, so $1-\lambda \sim 1/\sqrt{N}$), we get non-vanishing aggregate fluctuations. The problem is that empirically, such a large multiplier (of order of magnitude $\sqrt{N} \sim 10^3$) is very implausible: the impact of government purchases or trade shocks, say, would be much higher than we observe. Hence most economists do not see that "extremely large multiplier" Jovanovic route as plausible.

[5] This is done with shares $S_{it}/Y_{US,}$ for the year $t = 2000$. σ_i is the historical standard deviation of the growth rate of sales in firm i. Summing over the top 10 firms gives

$$\sigma_{GDP}^{\text{Micro}} = \left(\sum_{i=1}^{10} \sigma_i^2 \cdot \left(\frac{S_{it}}{Y_t}\right)^2\right)^{1/2} = 0.90\%$$

and over the top 100 firms gives $\sigma_{GDP}^{\text{Micro}} = 1.15\%$.

them with our initial microeconomic shocks, we can predict a large GDP volatility.

The general argument is the following: suppose that, for whatever reason (more on this later), it is generally true that the top J firms have an average size around 1% of GDP.

Proposition 1 *Suppose that across countries, the top J firms have each a share s of GDP, and have a volatility σ, and that the other firms make only negligible contributions to GDP volatility. Then (i) we get non-vanishing GDP fluctuations from micro shocks:*

$$\sigma_{GDP}^{Micro} = J^{1/2} s\sigma.$$

Proof. This was just show above. ■

Proposition 1 shows a very simple mechanism to generate non-vanishing aggregate fluctuations. Microeconomic fluctuations give rise to a non-trivial macroeconomic volatility.

It turns out that this view readily explains the scaling puzzle.

Proposition 2 *In addition to the assumptions of Proposition 1, suppose that the scaling relation (1) holds within countries, i.e. that a firm of size S has fluctuations $\sigma^{Firms}(S) \sim S^{-\alpha}$. Then we have*

$$\sigma_{GDP}^{Micro}(Y) \sim Y^{-\alpha}$$

so that firms and GDP have similar scaling.

Proof. Equation (1) says that $\sigma(S) = bS^{-\alpha}$ for some proportionality factor b. In a country of size Y, each of the top J firms has a size $S = sY$, so that its volatility is $\sigma = b(sY)^{-\alpha}$. Then, Proposition 1 gives $\sigma_{GDP}^{\text{Micro}} = J^{1/2} s^{1-\alpha} bY^{-\alpha} \sim Y^{-\alpha}$. ■

One can restate simply the intuition of the proof of Proposition2. Say that Japan's GDP is 3 times smaller than the US GDP. The top firms in Japan, as their typical size is roughly 1% of GDP, have a size typically 3 times smaller than the size of US firms. The Amaral et al. law $\sigma^{\text{Firms}}(S) \sim S^{-\alpha}$ implies that their volatility is roughly 3^{α} times (with $\alpha = .15$) bigger than the volatility of US firms. Given that GDP volatility is just the outcome of the volatility of the large firms, Japanese GDP volatility is 3^{α} larger than US GDP volatility. As Japan is 3 times smaller than the US, the scaling relationship $\sigma^{\text{GDP}}(Y) \sim Y^{-\alpha}$ holds.

The reader has now gotten the gist of the argument. We now look deeper at the mechanism. One payoff will be to get insight into the distribution of fluctuations. Still, the economic essence of the mechanism will be that the top firms will have a non trivial (or the order of magnitude of 1%) of GDP fluctuations, and the simple derivation in Proposition 2 is the essence of the mechanism.

2.4 Evidence on the concentration of economic activity

To see the impact of this, consider formula (3) when all firms have the same volatility σ :

$$\sigma_{GDP}^{\text{Micro}} = \sigma h \tag{4}$$

$$h = \left(\sum_{i=1}^{N} \left(\frac{S_{it}}{Y_t} \right)^2 \right)^{1/2} \tag{5}$$

The volatility of GDP is the volatility σ of firms, time the Herfindahl index h of the economy. The traditional Herfindahl is $H = h^2$, but for simplicity we will call h the Herfindahl. We condider the following measures:

$$h_S = \left(\sum_{i=1}^{N} \left(\frac{\text{Sales}_{it}}{Y_t} \right)^2 \right)^{1/2} \tag{6}$$

$$h_W = \left(\sum_{i=1}^{N} \left(\frac{\text{Workforce}_{it}}{\text{Total workforce}_t} \right)^2 \right)^{1/2} \tag{7}$$

Each measure has pros and cons. Throughout we make the approximation that the value added of a firm is proportional to its number of employees. In the polar case where firms do not use intermediary goods, the two Herfindahls above are identical. If there are intermediary goods, the h^S exceeds the true Herfindahl of the economy. Take an automobile firm: it will use rely on suppliers. So a shock to Ford has ripples as a shock to its suppliers.

We propose a simple economic model to modivate an "economic Herfindahl". Take an economy with initial GDP normalized at 1. Firms have chains of suppliers. The end firm j creates a value added V_j. Its immediate suppliers creates γV_j value added, with $\gamma < 1$, and we assume a self–similar structure where the supplier of rank i creates a value added

$$V_{ji} = (1 - \gamma)\, \gamma^i V_j. \tag{8}$$

The chain has a total value added:

$$\sum_{i \geq 0} V_{ji} = V_j$$

Suppose that shock happen for the whole product, with identical standard deviation σ :

$$\Delta V_{j,t+1} = \sigma V_{j,t}$$

As GDP is the sum of value added $Y_t = \sum_j V_{j,t}$, we have:

$$\begin{aligned}
\Delta Y_t &= \sum_j \Delta V_{j,t} \\
\left(\sigma_{GDP}^{\text{Micro}}\right)^2 &= var \frac{\Delta Y_t}{Y_t} = \sigma^2 \sum_j V_j^2
\end{aligned}$$

so the "economic" Herfindahl is:

$$h_E = \left[\sum_j V_j^2 \right]^{1/2}$$

as this gives $\sigma_{GDP}^{\text{Micro}} = \sigma h_E$. The econometrician does not observe directly h_E, as he does not observe the supply chain. Instead, he observes the sales of firm (i, j), which here take the expression:

$$S_{ji} = \sum_{i' \geq i} V_{j,i'} = \gamma^i V_j$$

and its share of value added V_{ji} given in (8). He then forms sales Herfindahls h_S and the workforce Herfindahl h_W as in (6)–(7). The next proposition shows how to infer from them the economic Herfindahl h_E.

Proposition 3 *Suppose that the econometrician has access only to the sales Herfindahl h_S and the workforce Herfindhal h_W. Then the economic Herfindahl follows:*

$$h_E = \left(2 h_W h_S - h_W^2\right)^{1/2}. \tag{9}$$

If firms have the same standard deviation σ, the macroecomic volatility is

$$\sigma_{GDP}^{Micro} = \sigma h_E. \tag{10}$$

Proof. We have:

$$\begin{aligned}
h_S^2 &= \sum_{i,j} S_{ji}^2 = \sum_j \left(\gamma^i\right)^2 V_j^2 \\
&= \frac{h_E^2}{1 - \gamma^2}
\end{aligned}$$

and

$$\begin{aligned} h_W^2 &= \sum_{i,j} V_{ji}^2 = \sum_j V_j^2 \left(\sum_i \left((1-\gamma)\,\gamma^i \right)^2 \right) \\ &= \frac{(1-\gamma)^2 \, h_E^2}{1-\gamma^2}. \end{aligned}$$

so

$$\frac{h_W}{h_S} = 1 - \gamma.$$

and

$$\begin{aligned} h_E^2 &= h_S^2 \left(1-\gamma^2\right) \\ &= h_S^2 \left(1 - \left(1 - \frac{h_W}{h_S} \right)^2 \right) \\ h_E &= \left(2 h_W h_S - h_W^2 \right)^{1/2}. \end{aligned} \tag{11}$$

■

We observe that as expected $h_W \leq h_E \leq h_S$.

We get our empirical Herfindahls from Worldscope. As Worldscope lists only the companies listed in the stock market, it will underestimate the true Herfindahls.

	All Countries	Rich Countries
h_W	3.0	3.4
h_S	10.9	19.2
h_E	7.5	10.9
$\sigma_{GDP}^{\text{Micro}}$	1.5%	2.2%

Table 1: Workforce Herfindahl, Sales Herfindahl, and economic Herfindahl, in percentage points, for the year 1999.
Rich countries are the countries with GDP greater than $13,000. Source: Worldscope.
For the induced GDP volatility, we take $\sigma_{GDP}^{\text{Micro}} = h^E \sigma$, with a volatility of firms $\sigma = 20\%$.

Taking a standard deviation $\sigma = 20\%$, which is rather a lower bound on microscopic volatility, we get:

$$\sigma_{GDP}^{\text{Micro}} = \sigma h^E. \tag{12}$$

Table 1 displays the results. We see that the economic herfindahl h^E is quite large: 7.5%. This corresponds to a GDP volatility

$$\sigma_{GDP}^{\text{Micro}} = 7.5\% \cdot 0.2 = 1.5\%$$

This is quite in the order of magnitude of GDP fluctuations. We conclude that GDP micro volatility is quantitatively large enough to explain macro volatility.

3 The distribution of fluctuations in firms and GDP growth

This section examines the prediction of the theory for the distribution of fluctuations in firms and GDP.

3.1 Preliminaries

3.1.1 Zipf's law for firms

Firm sizes (the sales, assets, or number of employees give the same results) in our model will be drawn from a power law distribution with exponent ζ close to 1 (but >1). This fact seems empirically true, as R. Axtell (1991) shows from a dataset with several million of firms. Figure **??** reproduces his finding.

Log frequency $\ln p(S)$ vs log size $\ln S$ of U.S. firm sizes (by number of employees) for 1997. OLS fit gives a slope of 2.059 (s.e.= 0.054; R^2 =0.992). This corresponds to a frequency $p(S) \sim S^{-2.059}$. Source: Axtell (2001).

Using the notation $p(S)$ for the frequency of firms of size S, we can do the regression:

$$\ln p(S) = a - (\zeta + 1) \ln S$$

and find an $R^2 = 0.992$, and a coefficient -2.059 ($\zeta = 1.059$), with a standard error of 0.054. So the probability density is close to $p(S) \sim S^{-(1+\zeta)}$, i.e. it is Pareto distributed with an exponent $\zeta = 1.059$. In the rest of the paper we will often take the approximation $\zeta = 1$.

The same phenomenon happens for cities (Zipf 1949), and the size (asset under management) of mutual funds (Gabaix et al. 2001). Gabaix (2001) proposes an explanation that generalizes Gabaix (1999) to entities with non-trivial birth and deaths. For[6] the rest of the paper, however, it suffices to take this power law distribution of the size of firms as having the power law distribution: $p(S) \sim S^{-(1+\zeta)} 1_{S>S_0}$(there must be a lower cutoff S_0 for sizes).

[6]In this paper, $f(S) \sim g(S)$ for some functions f, g, means that the ratio $f(S)/g(S)$ tends, for large S, to a positive real number. So f and g have the same scaling "up to a constant real factor".

3.1.2 Position of the problem

Amaral et al. and Canning et al. have another tantalizing finding. They plot the distribution of growth rates $\Delta \ln S_{it}$ of firms, and show that, after rescaling by $S^{-\alpha}$, firms of different sizes seem to have the same distribution of growth rate. We reproduce their finding in Figure 2 below. This is another example of what is called a "universal" relation in economics. We want here to explain this distribution.

We asked a second question. If the distribution of large firms is Zipf, rather than the "top firms have a non-infinitesimal fraction of GDP", do we get $\sigma^{\text{GDP}}(S) \sim S^{-\alpha}$ as in section 2?

We shall answer both questions at the same time.

3.2 Fluctuations without border effects: Distribution of firm growth

3.2.1 Distribution of growth rate when the units are Zipf, with no border effect

So a country of population H has firms drawn from a Pareto distribution with an exponent $\zeta > 1$. We draw successively $S_1, ...$So there are a number of $N = kH$, where k is some unimportant proportionality factor, so that

$$\sum_{i=1}^{N} S_i = Y$$

where Y, which is proportional to the population H, is the size of GDP.

Each firm i has an idiosyncratic shock, following the scaling law (1):

$$\frac{\Delta S_{it}}{S_{i,t-1}} = v S_{i,t-1}^{-\alpha} u_{it} \tag{13}$$

where u_{it} are independent random variables with mean 0 and variance $1, v$ is an index of the size of the shocks, and α is a number $[0, 1-\zeta/2]$. The case $\alpha = 0$ corresponds to Gibrat's law, the case $\alpha = 1/2$ corresponds to perfect diversification of firms. While several explanations of this fact can be given (see Amaral et al. 1998, Buldyrev 1997, Sutton 2001), we will not need to take a stand on them, and we will just take (1) as a given.

We can ask our two questions. When the subunits of an economic system follow a power law distribution with exponent ζ, and a volatility scaling $\sigma(S) \sim S^{-\alpha}$,what if the scaling of the aggregate entity? does its distribution of growth rate look like the empirical one?

In this section we talk about a "country" composed of many firms. However, a firm could as well be interpreted a small country, it self composed of many firms. This will allow us to derive a theory of the fluctuations of firms growth.

The GDP fluctuations are:

$$\Delta S_t = \sum_{i=1}^{N} \Delta S_{it} \tag{14}$$

$$= v \sum_{i=1}^{N} S_i^{1-\alpha} u_{it} \tag{15}$$

The key remark is that, as $S_i^{1-\alpha}$ has power law tails with an exponent $\zeta/(1-\alpha)$ which is < 2, the sum $\sum_{i=1}^{N} S_i^{1-\alpha} u_{it}$ has non-vanishing fluctuations. To see that more precisely, we form

$$m_t = \frac{\sum_{i=1}^{N} S_i^{1-\alpha} u_{it}}{N^{(1-\alpha)/\zeta}}$$

$$s_t = \frac{\sum_{i=1}^{N} S_i}{N}$$

$$g_t = \frac{m_t}{s_t^{(1-\alpha)/\zeta}} = \frac{\sum_{i=1}^{N} S_i^{1-\alpha} u_{it}}{\left(\sum_{i=1}^{N} S_i\right)^{(1-\alpha)/\zeta}}.$$

We have the following:

Proposition 4 *As $N \to \infty$, m_t and g_t tend in distribution to a symmetrical Levy stable distribution with exponent $\zeta/(1-\alpha) \leq 2$, and s_t to a constant real number.*

Proof. As $P(S_i > s) \sim s^{-\zeta}$, $P\left(S_i^{1-\alpha} > s\right) \sim s^{-\zeta/(1-\alpha)}$, so that $S_i^{1-\alpha}$ and $S_i^{1-\alpha} u_i$ have power law tails with exponent $\zeta/(1-\alpha) \leq 2$. So the Proposition is a direct consequence of Lévy's theorem saying that (e.g. Durrett 1996, p.153) for a sum of i.i.d. mean 0 random variables X_i, with (for $x \to +\infty$), $P(X_i > x)/P(X_i < -x) \to 1$, and $P(X_i > x) \sim x^{-\zeta'}$ for some k and $\zeta' \leq 2$, then

$$\frac{\sum_{i=1}^{N} X_i}{N^{1/\zeta'}}$$

converges to a symmetrical Levy distribution with exponent ζ'. The fact about s_t is just a statement of the law of large numbers, as $E[S_i] < \infty$. ■

The substantive economic interpretation is the following:

Proposition 5 *GDP fluctuations will be non-vanishing, and have the form:*

$$\frac{\Delta S_t}{S_t} = v S_t^{-\alpha'} g_t \tag{16}$$

with

$$\alpha' = \frac{\alpha + \zeta - 1}{\zeta} \tag{17}$$

and g_t is a symmetrical Levy stable distribution with exponent $\zeta / (1 - \alpha) \leq 2$. In particular, the volatility $\sigma(S)$ of GDP decreases with its size S, in a power law way:

$$\sigma^{GDP}(S) \sim S^{-\alpha'} \tag{18}$$

Proof. Direct, given (14) and the previous Proposition. ∎

The above did not depend on specific values of ζ. The following is a further consequence of the fact that, empirically, $\zeta \simeq 1$ (Axtell 2001, Gabaix 2001).

Proposition 6 *(Similar scaling of firms and countries). As $\zeta \simeq 1$ empirically, we have $\alpha' \simeq \alpha$, i.e. firms and countries should see their volatility scale with a similar exponent:*

$$\sigma^{Firms}(S) \sim \sigma^{GDP}(S) \sim S^{-\alpha}$$

Proof. Direct, from $\sigma^{\text{Firms}}(S) \sim S^{-\alpha}$, (17) and (18). ∎

3.2.2 Empirical evidence on the Levy distribution of firms fluctuations

It is also of interest to report the empirical distribution of the fluctuations of the growth of firms (from Amaral et al. 1997). We do this in Figure 2.

The model predicts that the distribution will have the shape of a symmetrical Levy distribution with exponent $1/(1-\alpha)$ with $\alpha = .15$. Figure 3 draws this distribution ($\ln p(x)$ vs x)

We see that the shapes are both much fatter than a Gaussian. We now investigate the best fist, assuming that the growth rate follows a symmetrical Levy distribution with exponent β. The Gaussian benchmark corresponds to $\beta = 2$.

Calling g_{it} the growth rate of firm i in year t, we transform $\gamma_{it} = A_t g_{it} + B_t$ such that for all t's, $E[\gamma_{it}] = 0$ and Median$(|\gamma_{it}|) = 1$. We plot the distribution of γ_{it}, which is strinkingly close to a Levy with exponent $1/(1-\alpha)$. There are some deviations, for very large $|\gamma|$. Hypothesizing

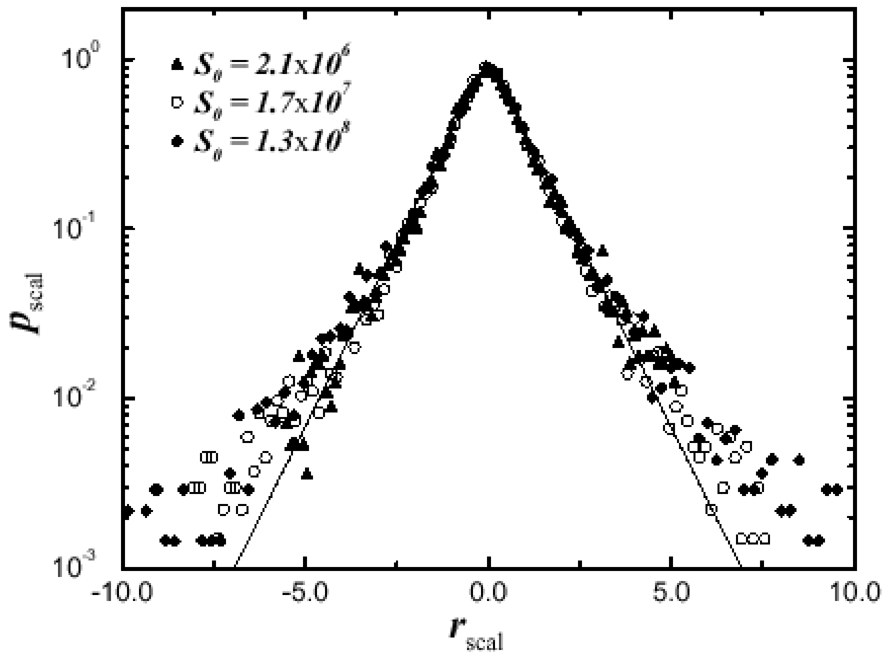

Figure 2: Empirical distribution of firms size fluctuations The shape is very similar to the one of the Levy distribution predicted by the model (see Figure 4 below). Source: Amaral et al. (1997).

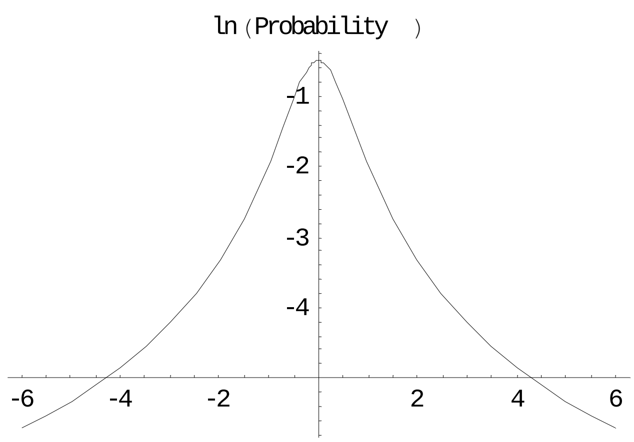

Figure 3: Log of distribution under the null of a Symmetrical Lévy distribution with exponent $1/(1-\alpha)$, with $\alpha = .15$.

that the γ_{it}, for $|\gamma_{it}| \leq \overline{\gamma}$ follow a Levy with exponent β, we estimate β by maximum likelihood. We take $\overline{\gamma} = 10$. As $P(|\gamma_{it}| \leq \overline{\gamma}) = 0.99$ empirically, this means that we fit the 99% of the points. We do this for each year separately, which give us a series of β. We find:

$$\begin{aligned} \text{Mean of } \beta &= 1.28 \\ \text{Standard deviation of } \beta_t &= 0.11 \\ \sigma(\beta) / (\text{Number of years})^{.5} &= 0.016. \end{aligned}$$

We conclude that empirically, $\beta = 1.28$ with a standard deviation of 0.016. The prediction is $1/(1-\alpha) = 1.18$ for $\alpha = 0.15$. The empirical data thus is fairly close to the theoretical prediction.

3.3 Fluctuations with border effects: Distribution of GDP growth

This subsection is more technical, and the reader can skip it in the first reading.

The above theory would not work for GDPs: the reason is that the typical top firm in a country is only a small fraction (say couple of percentage points) of a country's GDP. We need to modify the analysis to incorporate this fact. We adopt the following representation. The size of firms S_i are drawn from a power law with exponent $\zeta = 1 + \varepsilon$, but with bounded support $[1, mN]$.The density is assumed to by power law with an exponent ζ in $[1, mN]$, i.e.:

$$f(S) = \frac{\zeta}{1 - (mN)^{-\zeta}} S^{-\zeta - 1}$$

The total size is:

$$\begin{aligned} Y &= \sum_{i=1}^{N} S_i \\ &\simeq N/\varepsilon \end{aligned} \tag{19}$$

by the law of large numbers.

The object of this section is: what is the distribution of the fluctuations of such a system? This leads to a new type of distribution, which we call the modified Levy distribution, which is also "universal". The major payoff is that, if the size of the top firm is not allowed to be of more than a few percentage points of GDP, then the distribution of GDP is quite different: a generalized Levy distribution whose density is described in (26).

We define:

$$V_N := \frac{1}{N^{2-2\alpha}} \sum_{i=1}^{N} S_i^{2-2\alpha} \tag{20}$$

where S_i is drawn from the above distribution. We study V_N in the limit of large $N's$.We know, from the analysis above, that for $m = \infty$, V_N tends to a Levy distribution with exponent $1/(2-2\alpha)$.We study here its behavior for $m < \infty$. The tool of choice here is the Laplace transform (using $\zeta = 1 + \varepsilon \simeq 1$)

$$\begin{aligned} L^{V_N}(k) \quad &: \quad = E\left[e^{-kV_N}\right] = E\left[\exp \frac{-k}{N^{2-2\alpha}} \sum_{i=1}^{N} S_i^{2-2\alpha}\right] \\ &= \quad E\left[\exp \frac{-k}{N^{\gamma}} S_i^{\gamma}\right]^N \quad \text{with} \\ \gamma \quad &: \quad = 2-2\alpha \end{aligned} \tag{21}$$

Now

$$\begin{aligned} H \quad &: \quad = E\left[\exp \frac{-k}{N^{\gamma}} S_i^{\gamma}\right] \\ &= \int_1^{mN} \frac{\zeta}{1-(mN)^{-\zeta}} S^{-\zeta-1} \exp\left(\frac{-k}{N^{\gamma}} S_i^{\gamma}\right) dS \\ &= \frac{1}{1-(mN)^{-1}} \int_1^{mN} \exp\left(\frac{-k}{N^{\gamma}} S_i^{\gamma}\right) \frac{dS}{S^2} \\ &= \frac{1}{1-(mN)^{-1}} N^{-1} \int_{N^{-\gamma}}^{m^{\gamma}} \frac{\exp(-kt)}{\gamma t^{1+1/\gamma}} dt \text{ by the change in variables } S = N t^{1/\gamma} \end{aligned}$$

Note that as $N \to \infty$,

$$\begin{aligned} H \quad &\sim \quad N^{-1} \int_{N^{-\gamma}}^{m^{\gamma}} \frac{dt}{\gamma t^{1+1/\gamma}} \\ &\sim \quad 1 \end{aligned}$$

So we use (verifying that $H(k=0) = 1$)

$$\begin{aligned} H - 1 \quad &= \quad N^{-1} \int_{N^{-\gamma}}^{m^{\gamma}} \frac{\exp(-kt) - 1}{\gamma t^{1+1/\gamma}} dt + o\left(\frac{1}{N}\right) \\ &= \quad -\frac{1}{N} \psi(k) + o\left(\frac{1}{N}\right) \end{aligned} \tag{22}$$

with the new function:

$$\psi_{m,\gamma}(k) := \int_0^{m^\gamma} \frac{1-\exp(-kt)}{\gamma t^{1+1/\gamma}} dt \tag{23}$$

which has a closed form in terms of the Gamma function (analytically continued for $a < 0$): with

$$\Gamma(a,z) := \int_0^z e^{-t} t^{a-1} dt$$

we have:

$$\psi_{m,\gamma}(k) = -\frac{k^{1/\gamma}}{\gamma} \Gamma\left(-\frac{1}{\gamma}, k\ m^\gamma\right) - m \tag{24}$$

Finally, expressions (21) and (22) give, in the limit of large N's:

$$\begin{aligned} \ln L^{V_N}(k) &= N \ln H \\ &= N \ln \left(1 - \frac{1}{N}\psi(k) + o\left(\frac{1}{N}\right)\right) \\ &= -\psi(k) + o(1) \end{aligned}$$

Thus V_N converges in distribution to a well-defined random variable V, whose Laplace transform is:

$$L^V(k) = e^{-\psi(k)} \tag{25}$$

We can also establish the distribution of the fluctuations in Y :

Proposition 7 *If the subcomponents cannot have a size bigger that mN, for some finite m, the variance of Y scales as:*

$$\sigma_Y^2 \sim Y^{-2\alpha} V$$

where V is a random variable whose Laplace transform is:

$$L^V(k) := E\left[e^{-kV}\right] = e^{-\psi_{m,2-2\alpha}(k)}$$

where $\psi(k)$ is defined in (23)–(24). In the limit $m \to \infty$, V is a totally positive Levy distribution with exponent $1/(2-2\alpha)$.

In particular, all the moments are finite. Indeed one can easily calculate the cumulants of V (the κ_i such that $-\ln L^V(k) = \sum \kappa_i k^i/i!$), and find:

$$\kappa_i(V) = \frac{m^{\gamma i-1}}{\gamma i - 1}$$

Recall that e.g. the 4 first cumulants are: $(\kappa_i)_{i=1\ldots4}$ are respectively $\langle V\rangle, varV$, $\left\langle (V-\langle V\rangle)^3\right\rangle$, $\left\langle (V-\langle V\rangle)^4\right\rangle - 3varV$, i.e. the mean, variance, skewness and excess kurtosis.

We can also establish the distribution of the fluctuations in Y :

Proposition 8 *If the subcomponents cannot have a size bigger that mN, for some finite m, we have, given the standard deviations σ_i of a country, the fluctuations are normal*

$$\frac{\Delta Y}{Y} =^d Y^{-\alpha} V^{1/2} u$$

where u is a normal variables. In particular, if $m < \infty$, all moments are finite. Given only the size Y of the country, the fluctuations have the density:

$$f_{m,\alpha}(g) = \int_0^\infty e^{-\psi_{m,2-2\alpha}(k^2/2)} \cos(kg) \frac{dk}{\pi}. \tag{26}$$

and all the moments are finite. In the limit $m \to \infty$, this distribution tends to a symmetrical Levy distribution with exponent $1/(1-\alpha)$. In the limit $m \to 0$, this distribution tends to a gaussian.

Proof. $\frac{\Delta Y}{Y} =^d Y^{-\alpha} V^{1/2} u$ from above. So the Fourier transform of the fluctuations is:

$$\begin{aligned} F(k) &= E\left[e^{-ikV^{1/2}u}\right] = E\left[e^{-k^2V}/2\right] \\ &= e^{-\psi\left(k^2/2\right)} \end{aligned}$$

so taking the inverse Fourier transform we get (26).

When $m \to \infty$,

$$\begin{aligned} \psi_{m,\gamma=2-2\alpha}\left(k^2/2\right) &\to \int_0^\infty \frac{1-\exp\left(-k^2/2t\right)}{\gamma t^{1+1/\gamma}} dt = \frac{k^{2/\gamma}\Gamma(-1/\gamma)}{2^{1/\gamma}\gamma} \\ &= bk^{1/(1-\alpha)} \end{aligned}$$

for some b. The characteristic function is that of a symmetric Levy distribution.

When $m \to 0$,

$$\begin{aligned}\psi_{m,\gamma}(k) &= \int_0^{m^\gamma} \frac{1-\exp(-kt)}{\gamma t^{1+1/\gamma}} dt \\ &\sim \int_0^{m^\gamma} \frac{kt}{\gamma t^{1+1/\gamma}} dt \\ &= \frac{m^{\gamma-1}}{\gamma-1} k = \frac{m^{1-2\alpha}}{1-2\alpha} k\end{aligned}$$

so that $\psi_{m,\gamma}\left(k^2/2\right) \sim \frac{m^{1-2\alpha}}{1-2\alpha} k^2/2$, which shows that $\Delta Y/Y \left(\frac{m^{1-2\alpha}}{1-2\alpha}\right)^{-1/2}$ tends to a Gaussian(0,1) distribution. ■

When $m \to \infty$, there are no restriction on the support of the subunits, and we get the result in section, where we have a Levy $1/(1-\alpha)$ distribution. When $m \to 0$, even the largest firms are small (they are bounded above by $mY/\langle s \rangle$). So really even the total variance is the sum of lots of small variances, the central limit theorem applies, so that the fluctuations are gaussian. The proof shows that there order of magnitude is $m^{1/2-\alpha}$.

We choose the value of m the following way. For countries, the empirical top size of firms is roughly 2%. The top size of firms in the model is $m/\langle s \rangle$ times Y. So for calibrations we can take $m = 2\% \cdot \langle s \rangle = 0.5$ with $\langle s \rangle = 25$ employees.

For firms, the upper bound on the share of the "main unit" is 1, rather. So we get $m = \langle s \rangle = 25$. Numerical simulations that the resulting distribution is quite close, in the relevant domain, to the theoretical limit $m \to \infty$, so that we get a Levy with parameter $1/(1-\alpha)$.

3.3.1 Empirical evidence

The empirical distribution is plotted in Figure 4. Figure 5 shows the corresponding theoretical plot for the distribution of growth rates. We see that the two distributions are pretty close. (A formal measure of the distance will be put in the next iteration of the paper).

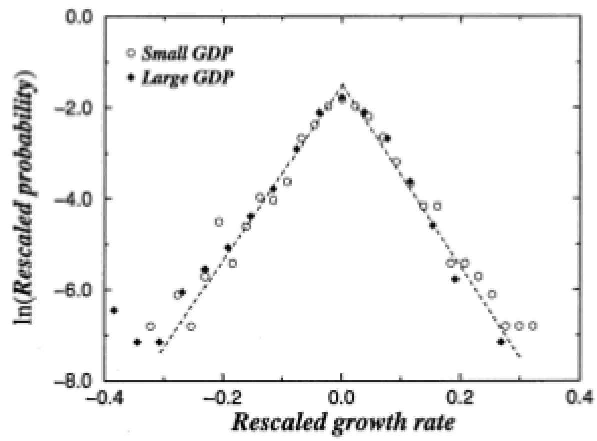

Figure 4: Empirical distribution of GDP fluctuations. Sources: Canning et al. (1998)

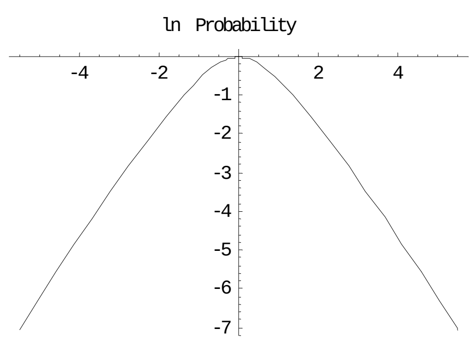

Figure 5: ln(Probability of a growth rate g) vs g under the null of the modified Levy distribution predicted by the model (with parameters $2-2\alpha = 1.7$ and $m = 1$).

4 Discussion

4.1 Extension of the model with feedback

The previous sections established the main theoretical results. Having calibration and greater descriptive realism in mind, we modify the model into:

$$\frac{\Delta S_{it+1}}{S_{it}} = \lambda \frac{\Delta S_t}{S_t} + v S_{i,t-1}^{-\alpha} u_{it} \tag{27}$$

The interpretation of the $\lambda \Delta S_t / S_t$ term is that there is a feed-back effect of past aggregate fluctuations ($\Delta S_t / S_t$) onto new decisions of firm i. This leads to a "multipliers" of shocks: a shock to firm j at time t affects . This feed-back could come from a variety of sources, among which (i) Long-Plosser (1983) production demand type (ii) Keynesian "aggregate demand" effects (iii) via the expectations (consumers, or businesses, see the other firms are doing very well, so they have more optimistic expectations and spend or invest more).

Having generality in mind, we allow firms specific shocks to be autocorrelated in an AR(1) manner:

$$u_{it} = \sum_{s \geq 0} \delta^s \varepsilon_{i,t-s}$$

where the ε_{it} are i.i.d.

So aggregate fluctuations are:

$$\begin{aligned} \Delta Y_t &= \sum_{i=1}^{N} \Delta S_{it} \\ &= \lambda \Delta Y_{t-1} + v \sum_{i=1}^{N} S_i^{1-\alpha} \sum_{s \geq 0} \delta^s \varepsilon_{i,t-s} \end{aligned}$$

thus, with L the lag operator ($L x_t = x_{t-1}$ for a random process x_t) :

$$\frac{\Delta Y_t}{Y^{1-\alpha}} = \frac{v}{Y^{1-\alpha}} \sum_{i=1}^{N} S_i^{1-\alpha} (1 - \lambda L)^{-1} (1 - \delta L)^{-1} \varepsilon_{it}$$

and if we look at the fluctuations sampled at horizon H (for instance, if the underlying unit of action is the quarter, and we look at yearly fluctuations, $H = 4$), defining:

$$\begin{aligned} \Delta S_t^{(H)} &= S_t - S_{t-H} \\ &= \left(1 + L + ... + L^{H-1}\right) \Delta S_t \end{aligned}$$

we get:

$$\frac{\Delta S_t^{(H)}}{S^{1-\alpha}} = \frac{v}{S^{1-\alpha}} \sum_{i=1}^{N} S_i^{1-\alpha} \eta_{it}$$

defining

$$\eta_{it} = \left(1 + L + ... + L^{H-1}\right)(1 - \lambda L)^{-1}(1 - \delta L)^{-1} \varepsilon_{it}$$

So the essence of this algebra is that, like in the simple case of section 2, we can represent:

$$\frac{\Delta S_t^{(H)}}{S^{1-\alpha}} = v \sigma g_t \tag{28}$$

with only with a messier expression for σ :

$$\sigma^2 = var\left(1 + L + ... + L^{H-1}\right)(1 - \lambda L)^{-1}(1 - \delta L)^{-1} \varepsilon_{it}$$

The main point is that, from (28) we get that $\Delta S/S$ has the fluctuations with the shape of g, it scales like $S^{-\alpha'}$, and (as is classic in the literature), the feedback λ can considerably increase the variance of aggregate fluctuations.

Given that the volatility of a firm is $var\left(1 + L + ... + L^{H-1}\right)(1 - \delta L)^{-1} \varepsilon_{it}$, the ratio

$$M = \left[\frac{var\left(1 + L + ... + L^{H-1}\right)(1 - \lambda L)^{-1}(1 - \delta L)^{-1} \varepsilon_{it}}{var\left(1 + L + ... + L^{H-1}\right)(1 - \delta L)^{-1} \varepsilon_{it}}\right]^{1/2} \tag{29}$$

plays the role of a "volatility multiplier". Indeed, we have:

$$\sigma_{\text{GDP}} = M \sigma_{\text{Micro}}, \text{ with} \tag{30}$$

$$\sigma_{\text{Micro}}^2 := \sum_i \sigma_i^2 \left(\frac{S_i}{Y}\right)^2 \tag{31}$$

where σ_i is the volatility of firm i, and S_i is size as a fraction of total GDP.

For $H = 1$, $\delta = 0$, we have

$$M = \frac{1}{\sqrt{1 - \lambda^2}}$$

For $H \gg 1/(1 - \lambda)$, $\delta = 0$, (no autocorrelation of shocks, but essentially all the propagation via $\lambda \Delta S_t / S_t$ happens within a period) we have:

$$M = \frac{1}{1 - \lambda}$$

As mentioned above, the nature of the feed-back leading to the multiplier could be very diverse: (i) Long-Plosser (1983) production demand type (ii) Keynesian "aggregate demand" effects (iii) via the expectations (consumers, or businesses, see the other firms are doing very well, so they have more optimistic expectations and spend or invest more). We do not want to take a stand here on the various "amplification mechanisms" proposed in macroeconomic research. We summarize their reduced form here by M. Given our earlier Compustat calibration $\sigma_{Micro} = 1.3\%$, it is not difficult to generate fluctuation $\sigma_{GDP} = M\sigma_{Micro}$ of empirical order of magnitude around 2%. We only need a multiplier around 1.5.

4.2 Time series properties of the model

Finally, the time-series shape of GDP fluctuations seems to correspond to the "hump-shaped" empirical one. If we estimated (on artificial data generated from our model) like e.g. Blanchard-Fisher (1989) an AR(2) for GDP, we would find the theoretical prediction of:

$$Y_t = 1.34Y_{t-1} - .41Y_{t-2} + \text{noise}$$

coefficients that are very close to empirical values of

$$Y_t = 1.25Y_{t-1} - .35Y_{t-2}$$

for US GDP.

4.3 Some highly speculative remarks on "fundamental" volatility

This subsection is highly speculative.

The above theory has the advantage of delivering the identical scaling of firms and countries. However, the reader might wince. There clearly are common shocks: oil shocks, world interest shocks, monetary policy shocks, exchange rate shocks etc. Why would the scaling still be preserved?

We propose the following view. Microeconomic shocks $\sigma_{\text{micro shocks}}$ define the "fundamental" volatility in the economy. In a large economy, say, the typical volatility, induced by micro shocks, will be say 2%/year. In a smaller economy, it will be 4%. Suppose that there is an aggregate shock, like an oil shock, bad news (a war outside the country), a high monetary policy shock. In the large economy, people are used to smallish reactions to news, and react by an amount proportional to 2%. Similarly, in the smaller economy, people will react by an amount proportional to 4%.

Call
$$\sigma_c^{\text{Nat, GDP}} = M\sigma_{\text{micro shocks}}$$
the "natural" volatility of the economy. Now, imagine that policy responses in a given country c are proportional to the "natural" volatility of economy c, and that policy mistakes are proportional too (policy makers "tremble" — say because of model uncertainty — by say 20%). This is, the variance of GDP (the aggregate policy shocks) will be proportional to the "natural" GDP volatility, i.e.
$$\sigma_c^{\text{Policy induced}} = k\sigma_c^{\text{Nat, GDP}}$$
for some constant of proportionality k. Then, the total variance of the economy will be:

$$\begin{aligned} \left(\sigma_c^{\text{Total}}\right)^2 &= \left(\sigma_c^{\text{Nat, GDP}}\right)^2 + \left(\sigma_c^{\text{Policy induced}}\right)^2 \\ \sigma_c^{\text{Total}} &= \sqrt{1+k^2}M\sigma_{\text{micro shocks}} \end{aligned} \tag{32}$$

So, as $\sigma_{\text{micro shocks}} \sim Y^{-\alpha}$, we would still have $\sigma_c^{\text{Total}} \sim Y^{-\alpha}$. Put simply, our "micro" shocks define the "natural" volatility of the economy, and the "modulus" of the typical reaction to shocks. Policy shocks, reacting to the natural volatility, are in magnitude proportional to it, and thus the total volatility is still proportional to the original "micro" shocks. Likewise, aggregate shocks create a reaction proportional to the natural modulus of the economy, e.g. a reaction proportional to $Y^{-\alpha}$.

This hypothesis can be tested (in a next iteration of the paper).

5 Related literature

5.1 Macroeconomics

It is useful to organize the themes. They are:

Macro-from-Micro: Micro shocks are enough to generate Macro shocks.

Equal Scaling: GDP fluctuations scale with size as firms growth fluctuations: $\sigma^{\text{Firms}}(S) \sim \sigma^{\text{GDP}}(S) \sim S^{-\alpha}$. A weak version of (Equal Scaling) is:

Negative Slope: GDP fluctuations decline with size. $\sigma^{\text{GDP}\prime}(S) < 0$.

A few papers have proposed way to generate macro shocks from purely micro shocks. The earliest may be Jovanovic (1987), which we discussed in section. As it relies on an extremely large multiplier ($M \sim N^{1/2}$, so that M has the order of magnitude of 1000), we do not view it as plausible.

Another route is the one of self-organizing criticality explored in a very innovative paper by Bak et al. (1993) and Scheinkman, J.A., Woodford (1994). While we have much sympathy for their approach (which is very different from ours), their model generates what is probably "too fat tailed" fluctuations: they have an exponent of 1/3, so that fluctuations don't even have a mean, much less a variance. Nirei (2001) proposes a elaborate model whose spirit is related to Bak et al. 1993, and finds fluctuations with a power law exponent 1/2. His model has enough free parameter to accommodate (at the price of a high elasticity of labor supply) any prediction for the scaling of GDP fluctuations with size.

Horvath (1998, 2000) explores the possibility that sectoral shocks might be enough to account for GDP fluctuations. It is quite empirical in nature. This relies on the sparsity of the input-output matrix, but one does not understand why it is sparse. There is no prediction about Equal scaling or Negative slope.

Durlauf (1993)'s interesting paper gets some Macro-from-Micro. It does not generate Equal Scaling or (Negative slope).

Acemoglu and Zilibotti (1997) have a model where large countries have a smaller volatility. Their mechanism is very different from the present one. It predicts Negative Slope, but not Equal Scaling. It does not use Macro to Micro.

5.2 Some other power laws in economics

Incomes (Pareto 18xx).

A number of economic systems appears to follow Zipf's law, where the probability that an entity is $> S$ is proportional to $1/S$: cities, firms, mutual funds, web sites. Gabaix (1999) provides an explanation and a survey of the literature.

Gabaix et al. (2003) present a series of puzzling facts on the distribution of stock market returns, and an explanation. We have $P(r_t > r) \sim P(r_t < -r) \sim r^{-3}$ for r between 1 and 80 standard deviations of returns. Crashes do not appear to be outliers to this distribution. They base their explanation on another Zipf's law, for mutual funds: those extreme returns are due to the trading behavior of large market participants.

6 Conclusion

Obviously there are other, "macro" shocks: monetary policy shocks, policy shocks, trade (e.g. exchange rate) shocks, and possibly aggregate productivity shocks. However, is it possible that, though they are the most visible ones, they are not the major contributors to GDP fluctuations.

This paper makes a theoretical case for the possibility that purely micro shocks are an important drivers of GDP fluctuations. It presents a simple calibration, showing that micro shocks indeed are a quantitatively plausible source of aggregate fluctuations. The model predicts several other features that are borne out in the data: It predicts that large countries have smaller volatility than small countries with a power law relationship identical to that found for firms (countries with a GDP of S have a volatility proportional to $S^{-\alpha}$ with $\alpha \simeq .15$). It delivers the distribution of GDP fluctuations close to the one found empirically (a truncated Lévy distribution), and the time-series properties (hump-shaped impulse response) of deviations from trend. It also predicts that the distribution of firms growth rates will be close to a Levy distribution with exponent $1/(1-\alpha)$, which appears in pretty good agreement with the empirical distributions.

The present paper, at least, lays down the theoretical possibility that those micro shocks are an important, and possibly the major, part of the origins of business cycle fluctuations.

7 Appendix A: Lévy distributions

A symmetrical Lévy distribution with exponent ζ has the distribution:

$$\lambda(x,\zeta) = \frac{1}{\pi}\int_0^\infty e^{-k^\zeta}\cos(kx)\,dk \tag{33}$$

and the cumulative:

$$\Lambda(x,\zeta) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty e^{-k^\zeta}\frac{\sin(kx)}{k}dk \tag{34}$$

Its asymptotic property is that $P(X > x) \sim x^{-\zeta}$. ζ has to be ≤ 2.

Unfortunately, there is not closed form formula for λ and Λ except in the case $\zeta = 1$ (Cauchy distribution) and $\zeta = 2$ (normal distribution).

Weron and Weron (1995) offer a review of the Monte Carlo generation of Levy random variables.

8 Appendix B: Calculating the maximum likelihood estimate of the exponent ζ for a truncated Levy distribution

We take only the observations $|x_i| \leq M$ for some M.

The log-likelihood is:

$$L(x,\zeta) = \ln\frac{\lambda(x,\zeta)}{\int_{|y|\leq M}\lambda(y,\zeta)\,dy} = \ln\lambda(x,\zeta) - \ln(1 - 2\Lambda(-M,\zeta))$$

because $\lambda(x,\zeta)$ is even in x.

We will optimize the log-likelihood over ζ, i.e. find ζ s.t.

$$\frac{1}{I}\sum_{i=1}^{I} L_\zeta(x_i,\zeta) = 0$$

with

$$L_\zeta(x,\zeta) = \frac{\lambda_\zeta(x,\zeta)}{\lambda(x,\zeta)} + \frac{2\Lambda_\zeta(-M,\zeta)}{1 - 2\Lambda(-M,\zeta)}.$$

Here

$$\begin{aligned}
\frac{\lambda_\zeta(x,\zeta)}{\lambda(x,\zeta)} &= \frac{\int_0^\infty e^{-k^\zeta}k^\zeta \ln k\cos(kx)\,dk}{\int_0^\infty e^{-k^\zeta}\cos(kx)\,dk} \\
\frac{2\Lambda_\zeta(-M,\zeta)}{1 - 2\Lambda(-M,\zeta)} &= \frac{\int_0^\infty e^{-k^\zeta}k^\zeta \ln k\frac{\sin(kM)}{k}dk}{\int_0^\infty e^{-k^\zeta}\frac{\sin(kM)}{k}dk}
\end{aligned}$$

Because of the computational burden of calculating the above expressions, we use interpolations. Call

$$h(x,\zeta) = \frac{\lambda_\zeta(x,\zeta)}{\lambda(x,\zeta)}.$$

We shall fit, for $m = 8$, and

$$\{\zeta_0, ..., \zeta_m\} = \{1, 1.1, ..., 1.8\}$$

a polynomial in ζ of degree m:

$$\widehat{h}(x,\zeta) = \sum_{i=0}^{m} A_i(x)\,\zeta^i.$$

To do this, we calculate numerically $h(x,\zeta_i)$ for all x in a very fine grid[7], and then fit the $A_i(x)$ s.t. $\widehat{h}(x,\zeta_i) = h(x,\zeta_i)$ for all $i = 0, ..., m$. This leads to:

$$\widehat{h}(x,\zeta) = \left(\zeta^i\right)'_{i=0,\dots,m}\left(\left(\zeta_i^j\right)_{i,j=0,\dots,m}\right)^{-1}\left(h\left(x,\zeta_j\right)\right)_{j=0,\dots,m}. \qquad (35)$$

So the MLE estimator $\widehat{\zeta}$ is the solution of $H\left(\widehat{\zeta}\right) = 0$, for

$$H(\zeta) := \frac{1}{I}\sum_{i=1}^{I}\widehat{h}(x_i,\zeta) + \frac{2\Lambda_\zeta(-M,\zeta)}{1 - 2\Lambda(-M,\zeta)}$$

We calculate the standard error of $\widehat{\zeta}$ by Monte-Carlo simulations.

9 Appendix C: Taking into accounts upper limits on sizes, and the resulting distribution of GDP growth

As a calibration, take $\lambda = 1/2$ (so the pure multiplier is $1/(1-\lambda) = 2$), $\delta = 2^{1/4} = .84$ (so the half-life of an idiosyncratic shock is 1 year), and $h = 4$ (shocks happen at the quarterly frequency, we look at yearly standard deviation of GDP). Using equation (29) leads to $M \simeq 2$. we get, for the volatility of annual GDP:

[7] In our implementation, *Mathematica*'s Interpolate function chose this very fine grid.

$$\sigma_{GDP} = 2 \cdot 1.2\% = 2.4\%.$$

We conclude that the "micro shocks" model, combined with a multiplier of only $1/(1-\lambda) = 2$, generates aggregate shocks of a magnitude comparable to those observed in actual business cycles.

9.1 An abstract calibration

We will take for the distribution of firms, for $s \geq 1$:

$$P(S > s) = s^{-\zeta} \text{ with } \zeta = 1 + \varepsilon$$

so that the minimum size (in number of employees) of a firm is 1, and the mean size is:

$$\langle S \rangle = \int_1^\infty s \cdot \zeta \cdot s^{-\zeta-1} ds = \frac{\zeta}{\zeta - 1} = \frac{1}{\varepsilon} + 1 \simeq \frac{1}{\varepsilon}$$

Matching US values of $\langle S \rangle = 21$, we get:

$$\varepsilon = .05.$$

More generally, the theory (Gabaix 2001b) predicts only $\varepsilon \simeq 0^+$, so there is a certain arbitrariness in the value of ε. We view this values of ε as reasonable.

Take an economy of the US size. So the total size, of GDP is (in workers equivalent):

$$Y = 10^8.$$

So the total number of firms will be:

$$N = \frac{Y}{\langle S \rangle} \simeq \varepsilon Y.$$

So we find the value of $N \simeq 5$ million.

We know that, in the upper tail, the volatility satisfies (**??**).To extend this to the lower tail of the distribution, we assume that the volatility saturates (this assumption, as we shall see later, does not matter for our result) for sizes below a certain S^* (from Amaral et al. 1997, we know $S^* \leq 500$ employees)

$$\sigma(S) = v \max\left(S^{-\alpha}, S^{*-\alpha}\right) \tag{36}$$

So the GDP volatility will be (with no multiplier, i.e. $M = 1$)

$$\begin{aligned} \Delta S_t &= \sum_{i=1}^{N} \Delta S_{it} \\ &= \sum_{i=1}^{N} S_{it}\sigma\left(S_{it}\right) u_{it} \end{aligned}$$

Now, even with the truncation, $S_{it}\sigma\left(S_{it}\right)$ is a variable asymptotically equal to $vS_{it}^{1-\alpha}$ for large S_{it}'s, so that Levy's theorem applies, and we have:

$$\Delta S_t \sim N^{1-\alpha} v g_t$$

with g_t a standard Levy with exponent $\zeta' \simeq 1.15$. So the average value of the fluctuations is:

$$\begin{aligned} E\left[\left|\frac{\Delta S_t}{S_t}\right|\right] &= \frac{N^{1-\alpha}v}{N/\varepsilon} E\left[|g|\right] \\ &= N^{-\alpha} v E\left[|g|\right] \end{aligned} \quad (37)$$

We get the following values:

- For $\zeta = 1.15$, and g symmetrical standard Levy with exponent ζ, $E\left[|g|\right] = 4.7$ (evaluated numerically).

- $N = 5 \cdot 10^6$, so that $N^{-.15} = .046$

-v is such that a firm of size $S = 10^4$ has a volatility around $\sigma = vS^{-2}$ =25%, so that:

$$v = \frac{.25}{\left(10^4\right)^{-.2}} = 0.99$$

So we conclude:

$$\begin{aligned} E\left[\left|\frac{\Delta S_t}{S_t}\right|\right] &= N^{-\alpha} v E\left[|g|\right] \\ &= \left(5 \cdot 10^6\right)^{-.2} \cdot 0.99 \cdot 4.7 \\ &= 1.7\% \end{aligned}$$

So we expect that, even with no multiplier, we will find fluctuations around 1.7%. This has already the right order of magnitude. With a multiplier of size M, we will find:

$$E\left[\left|\frac{\Delta S_t}{S_t}\right|\right] = M \cdot 1.7\%.$$

With $\alpha = .15$, doing the same calculation (and with $E[|g|] = 5.7$ for an exponent of 1.15), we would find:

$$E\left[\left|\frac{\Delta S_t}{S_t}\right|\right] = M \cdot 2.8\%.$$

So here again we do not need a multiplier: $M = 1$ (i.e. no multiplier) gives already a large enough aggregate GDP volatility.

10 Appendix D: Herfindahls

If we have, for firm i :

$$\Delta S_{it}/S_{it-1} = \sigma_i u_{it}$$

with u_{it} has finite variance, and $\sigma_i \sim S_i^{-\alpha}$.

For GDP we have:

$$\begin{aligned} \Delta Y/Y &= \sum_i \frac{\Delta S_{it}}{S_{it}} \frac{S_{it}}{Y} \\ &= \sum_i \sigma_i u_{it} \frac{S_{it}}{Y} \end{aligned}$$

so we should have

$$var \Delta Y/Y = V^2 \quad (38)$$

$$V^2 := \sum_i \sigma_i^2 \left(\frac{S_{it}}{Y}\right)^2 \quad (39)$$

If we don't observe σ_i perfectly, we can do:

$$\sigma_i = S_i^{-\alpha} v_i$$

for some random variable v_i, so that:

$$\begin{aligned} E\left[(\Delta Y/Y)^2 \mid (v_i)_{i=1...N}\right] &= \sum_i \left(S_i^{-\alpha} v_i\right)^2 \left(\frac{S_{it}}{Y}\right)^2 \\ &= Y^{-2\alpha} \sum_i v_i^2 \left(\frac{S_{it}}{Y}\right)^{2-2\alpha} \end{aligned}$$

To analyze the scaling of a sum of random variables $\sum_i v_i^2 \left(\frac{S_{it}}{Y}\right)^{2-2\alpha}$, we have to consider two cases.

If the v_i have finite variance (e.g. if the growth rate of firms has finite variance), we get

$$\begin{aligned} E\left[(\Delta Y/Y)^2\right] &= E\left[Y^{-2\alpha}\sum_i v_i^2\left(\frac{S_{it}}{Y}\right)^{2-2\alpha}\right] \\ &= Y^{-2\alpha}H^2 \end{aligned}$$

with

$$H^2 = \sum_i \left(\frac{S_{it}}{Y}\right)^{2-2\alpha}$$

the modified Herfindahl$(2-2\alpha)$.We write this:

$$\sigma_{\Delta Y/Y} \sim Y^{-\alpha}H. \tag{40}$$

- If the v_i have infinite variance: take the case where v_i^2 are iid a totally positive Levy with exponent z. Iin the micro theory of the paper (Proposition 5, interpreted for firms) , we have $z = z^*$ for

$$z^* = \frac{1}{2-2\alpha} \tag{41}$$

We use the fact

Proposition 9 *If $X_1,,,X_n$ are iid standard Levy's with the parameters (M_i,κ,ζ), and $C_1,...,C_n$ are real, then $X = \sum_{i=1}^n C_iX_i$ is also a Levy, with exponent(κ,ζ) and scale parameter:*

$$M = \left(\sum (C_iM_i)^\zeta\right)^{1/\zeta}.$$

In other terms, if X_i scale like M_i, then $X = \sum_{i=1}^n C_iX_i$ scales like $\left(\sum (C_iM_i)^\zeta\right)^{1/\zeta}$. Note that we find the well-known result of Gaussian distributions for $\zeta = 2$.

As an application, if v_i^2 is are iid Levy with exponent z, then $(\Delta Y/Y)^2$ can be written:

$$\sigma^2_{\Delta Y/Y} = Y^{-2\alpha}\left(\sum_i \left(\frac{S_{it}}{Y}\right)^{z(2-2\alpha)}\right)^{1/z}\Lambda \tag{42}$$

where Λ is a Levy with exponent z.

In the case $z = 2$ (Gaussian case), then we find (40), and Λ is a gaussian.In the polar case of the micro-model in the paper, with $z = z^*$ in (41), then

$$\begin{aligned}\sigma^2_{\Delta Y/Y} &= Y^{-2\alpha}\left(\sum_i \left(\frac{S_{it}}{Y}\right)^1\right)^{1/z}\Lambda \\ &= Y^{-2\alpha}\cdot 1\cdot\Lambda\end{aligned}$$

(as $\sum_i S_{it}/Y = 1$) so that, somewhat surprisingly, there is *no relation* between $\sigma_{\Delta Y/Y}$ and a micro-Herfindahl.

11 Appendix D: Evidence on the scaling law of growth rates

11.1 Microeconomic scaling

The scaling law says that a unit with size S, in a year t, will have a standard deviation:

$$\sigma(S,t) = \text{standard deviation}\left(\ln S_{t+1} - \ln S_t \mid S_t = S\right) = b_t S^{-\alpha_t} \tag{43}$$

Amaral et al. (1997) present evidence for the scaling law for a particular year t. We extend here their empirical analysis.

We first proceed with size as a measure of sales. We estimate α_t for each year, and plot in Figure XX the resulting values of α_t. We show here that α_t has remained fairly constant throughout the years. Its mean value is 0.188.

Interestingly, the coefficient b_t has increased over the year.

We have estimated α for the firms in different SIC 1-digit codes. The coefficient is constant across 1-digit industries.

12 Appendix E: The maths with thin tailed distributions of firm sizes

We first briefly recall the reason why, in macroeconomics, one usually appeals to common (or at least sector-wide) aggregate shocks. With about $N = 10^6$ firms, one would expect the sum of their idiosyncratic shocks to be vanishingly small. Indeed, say that GDP has a size Y, and is composed

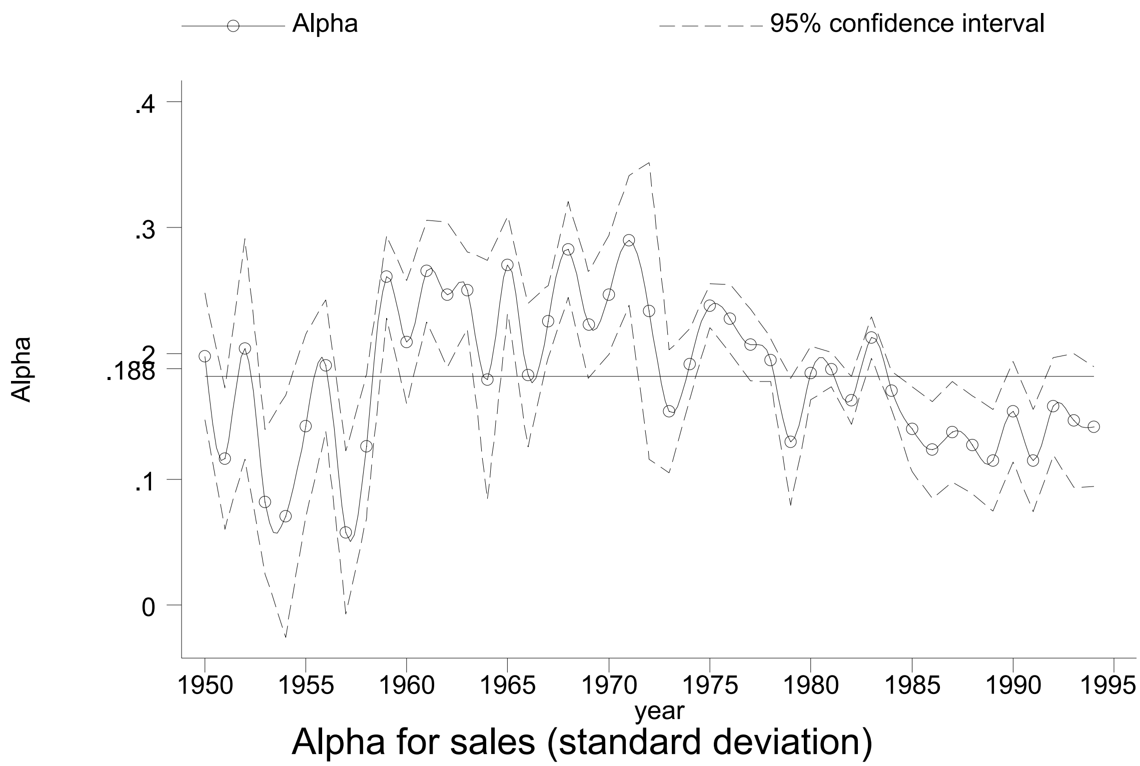

Figure 6: Time series of the scaling exponent α_t for the growth of sales. For each year t we estimate the scaling exponent α_t, such that $\sigma\left(g_t \mid S_t = S\right) \sim S^{-\alpha_t}$.

of N firms of size $S_{i,t=0}$ i.i.d. (For a large economy like the US, the order of magnitude is $N \simeq 10^6$). GDP is the sum of the total productions, so that:

$$Y_t = \sum_{i=1}^{N} S_{it} \tag{44}$$

Hence the total number of firms is, to a good approximation:

$$N \simeq E[N] = \frac{Y}{\langle S \rangle}$$

Suppose each firm has a production shock of say $v = 25\%$ annual, i.e., with a mean initial size $E[S_{i,t=0}]$, we have:

$$\frac{\Delta S_{it}}{S_{it}} = v\varepsilon_{it}$$

where the ε_{it} are i.i.d. variables of variance 1. Then, as total GDP is the sum of individual outputs:we have, for GDP fluctuations:

$$\Delta Y_t = \sum_{i=1}^{N} \Delta S_{it} = v \sum_{i=1}^{N} S_{it}\varepsilon_{it}$$

so that

$$var \Delta Y_t = v^2 \sum_{i=1}^{N} var\left(S_{it}\varepsilon_{it}\right) = v^2 N \left\langle S^2 \right\rangle$$

and (**??**) giving $Y = N \langle S \rangle$,

$$var\left(\frac{\Delta Y_t}{Y_t}\right) = \nu^2 \frac{\left\langle S^2 \right\rangle}{\langle S \rangle^2 N} \tag{45}$$

So aggregate fluctuations $\sigma\left(\Delta Y_t / Y_t\right)$ decay in $1/\sqrt{N}$. The conclusion is that micro fluctuations (the ε_{it}) cannot explain the macro fluctuations: for $N = 10^6$, and firms of identical size so that $\left\langle S^2 \right\rangle / \langle S \rangle^2 = 1$ and say $v = 25\%$ in annual value, we get:

$$\sigma\left(\frac{\Delta Y_t}{Y_t}\right) = \frac{v}{\sqrt{N}} = \frac{25\%}{10^3} = .025\%/\text{year}$$

which is just too small to account for the empirical size of macroeconomic fluctuations.

A multiplier (more later on its later) might increase the impact of each shock. If the model becomes:

$$\frac{\Delta S_{it}}{S_{it}} = \lambda \frac{\Delta S_t}{S_t} + v\varepsilon_{it}$$

the total impact of a unitary shock is

$$M = 1/\left(1-\lambda\right),$$

and the total long run annualized volatility is:

$$\sigma\left(\frac{\Delta Y_t}{Y_t}\right) = \frac{1}{1-\lambda}\nu\frac{\left\langle S^2\right\rangle^{1/2}}{\left\langle S\right\rangle\sqrt{N}} \tag{46}$$

Thus a multiplier $M = 1/\left(1-\lambda\right)$ will increase the predicted fluctuations from around 0.025%/year to M times that value. One route out of this has been taken by Jovanovic (1987), who observes that the multiplier is very large $(1/\left(1-\lambda\right) = M \sim \sqrt{N}$, so $1-\lambda \sim 1/\sqrt{N})$, we get non-vanishing aggregate fluctuations. The problem is that empirically, such a large multiplier (of order of magnitude$\sqrt{N} \sim 10^3$) is very implausible: the impact of government purchases or trade shocks, say, would be much higher than we observe. Hence most economists do not see that "extremely large multiplier" Jovanovic route as plausible.

However, formulae (45) and (46) suggest another route. If the distribution of firms has fat tails, so that $\left\langle S^2\right\rangle = +\infty$ (at least as a mathematical approximation), then the formula (45) for $\sigma\left(\Delta Y_t/Y_t\right)$ becomes infinite, and the $1/\sqrt{N}$ degeneracy becomes irrelevant. This route seems a priori promising, because empirically the size distribution of firms does have fat tails, and the best statistical description leads to $\left\langle S^2\right\rangle = +\infty$. We turn to this, and its consequences, in the next section.

Figure XXX plots the values of $\ln b_t$ in (43). One sees that $\ln b_t$ increases roughly linearly, at a mean rate 2.5% per year. This increase in microeconomic volatility is quantitatively identical if one take the number of employees as a measure of size.

13 Appendix F: A simple model illustrating the mechanics of the paper

The paper presents a mechanism, that emerge from a variety of economic structures. We present here one possible type of model that generates the mechanism. Markets are competitive. Firm i has a capital K_{it}. It invests in a technology with random productivity A_{it} such that $E[A_{it}]$ is constant across i's and

$$\sigma(A_{it}) = bK_{ti}^{-\alpha} \tag{47}$$

A variety of mechanisms (e.g. Amaral et al. (1998), Sutton (2001)) can generate the microeconomic scaling (47). They typically consider that the firm of size S is made up of N smaller units, with $N \sim S^{\beta}$, wich generates (1) and (47) with $\alpha = \beta/2$. Capital is fully reinvested, so that:

$$K_{i,t+1} = A_{i,t+1}K_t \tag{48}$$

GDP is simply:

$$Y_t = \sum_i A_{i,t}K_{t-1}.$$

Adding labor does not change the conclusion of this paper. Suppose that the production function is $A_{i,t}F(K_{ti}, L_{ti})$, with constant returns to scale. Risk neutral firms maximize

$$\max_{L_{it}} E[A_{it}]\,F(K_{ti}, L_{ti}) - w_t L_{ti}$$

the quantity of labor chosen L_{ti}will be $L_{it} = \lambda_t K_{it}$, for a factor of proportionality λ, so that we will have:

$$K_{i,t+1} = A_{i,t+1}F(K_{it}, \lambda_t K_{it}) - w_t\lambda_t K_{it} = (A_{i,t+1}F(1,\lambda_t) - w_t\lambda_t)\,K_{it}$$

so that the equation of motion follows the same structure as (48), with random productivity:

$$A'_{it} = A_{i,t+1}F(1,\lambda_t) - w_t\lambda_t.$$

GDP is

$$Y_t = \sum_i A_{i,t}K_{t-1}F(1,\lambda_{t-1}).$$

so that it evolves as the stochastic sum in the paper.

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