Zipf's Law of City Sizes: A Microeconomic Explanation *Far From Equilibrium*

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Motivations

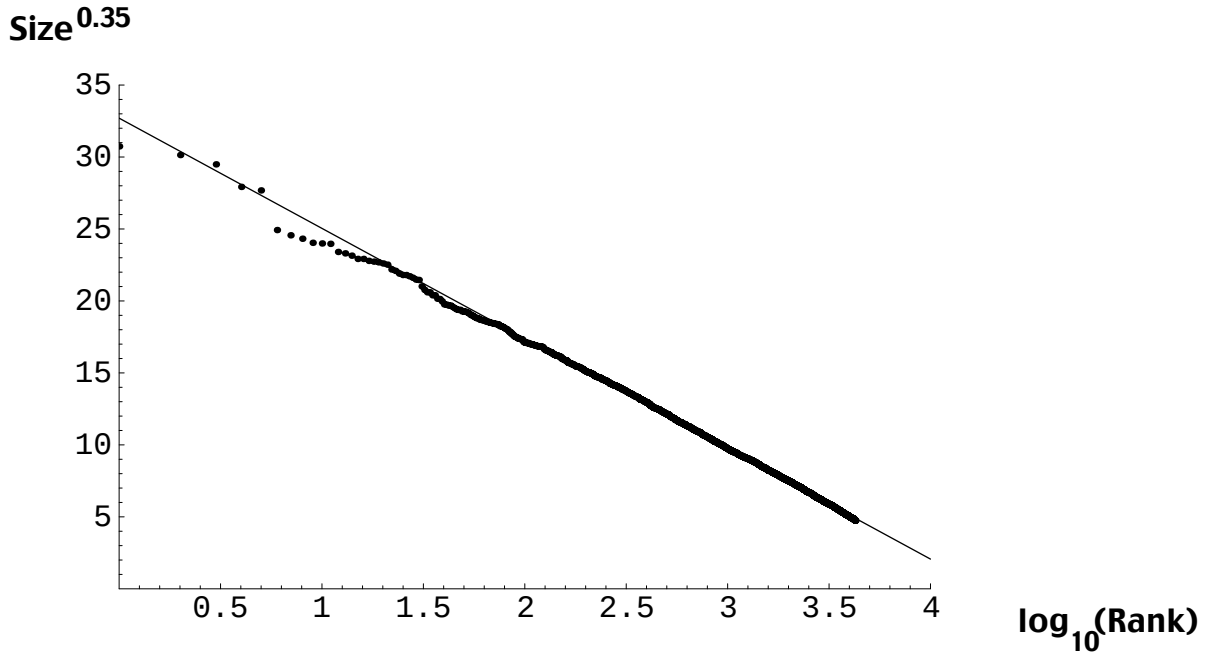
- *Positive* goals:
  - Data on cities and firms are unique in the social sciences:
    - » Regular: simple functional forms
    - » Stable: Invariant over 100–200 years
    - » Robust: Across regions, countries
  - Today there exists no microeconomic explanation for these data
- *Normative* goals:
  - Governments at all levels want more economic growth
  - Are they providing the right incentives (e.g., tax breaks)?
  - Each locality wishes to nucleate its own industrial clusters
  - Same ideas might be leveraged for development worldwide

Size Distribution of U.S. Firms, 1997

Approximately 100 million workers and < 4 million firms in U.S.:

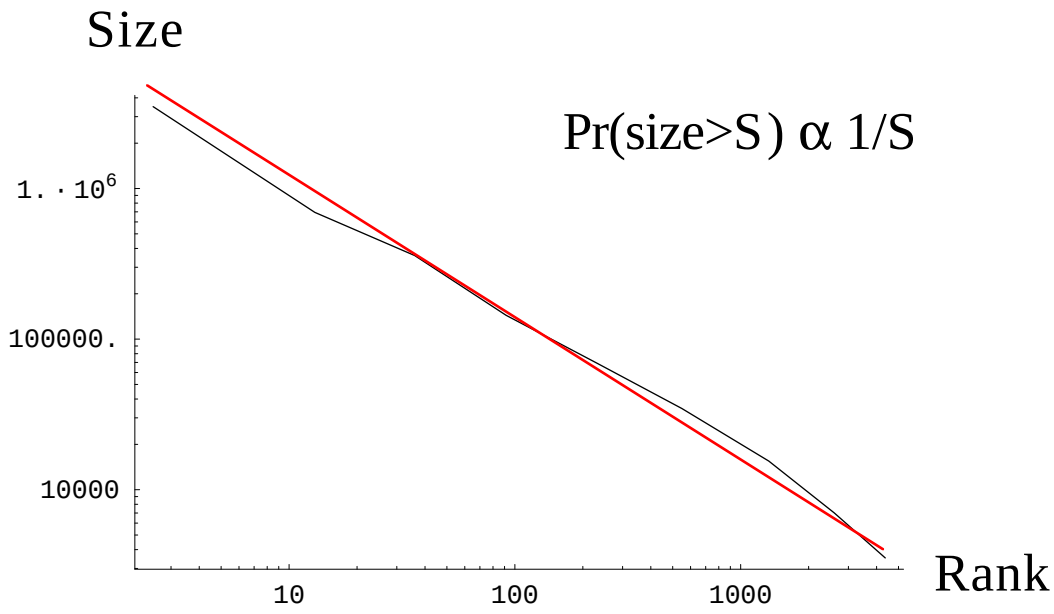
- **median firm size is < 10, average firm size is ~25**
- **median worker is employed at a firm of size 100**
- **~80% of employees work in firms smaller than 500**

Firm Sizes are Weibull Distributed



The Weibull is an extreme value distribution

Zipf's Law: U.S. in 1960



- **Valid in the U.S. for at least 150 years, despite changes in the number and average size of cities**
- **Valid in most industrial countries (not Russia!), save 'king' cities**
- **Robust to changes in rank (serial correlation in growth rates)**
- **Other city facts: wages proportional to city size$^{0.06-0.08}$**

Why a New Approach is Needed

- Neoclassical economics:
  - Behavioral model for people:
    - » Fully-informed
    - » Rational
  - People interact only indirectly with one another (through markets)
  - Focus on equilibrium outcomes
- Complexity approach:
  - People are adaptive
  - They interact directly with one another
  - Focus on dynamics
  - Methodology is agent-based modeling

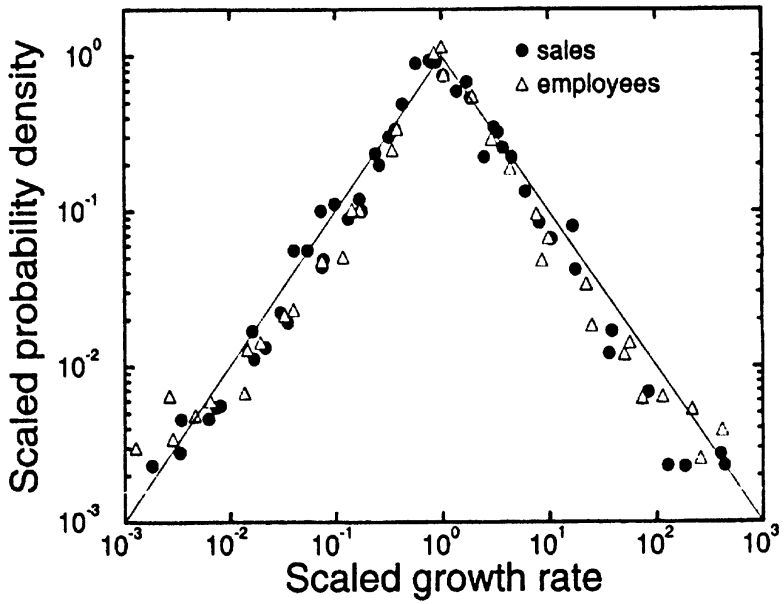
The Complexity Approach

- **Agent Computation:**
  - Create a population of agents in software and give them rules of interaction
  - 'Spin' the society forward in time
  - Study what emerges
- **Firms:**
  - Can we get firms to self-organize?
  - What rules of interaction lead to skewed distributions of firm sizes?
- **Cities:**
  - Can we get agents and firms to agglomerate?
  - What rules lead to Zipf-like agglomerations?

Many Theories of the Firm

- Textbook orthodoxy: Firms as black boxes
  - Production function specifies technology
  - Profit maximization specifies behavior
  - Winter's critique: Not even methodologically individualist
- Coase and Williamson ('New Institutionalism'):
  - "Transaction cost" approach
- Principal–Agent approaches:
  - Firm as nexus of contracts (incomplete contracts)
- Firm as Information Processing Network
- Evolutionary economics:
  - Purposive instead of maximizing behavior
- Industrial Organization
  - Empirical studies have little connection to theory

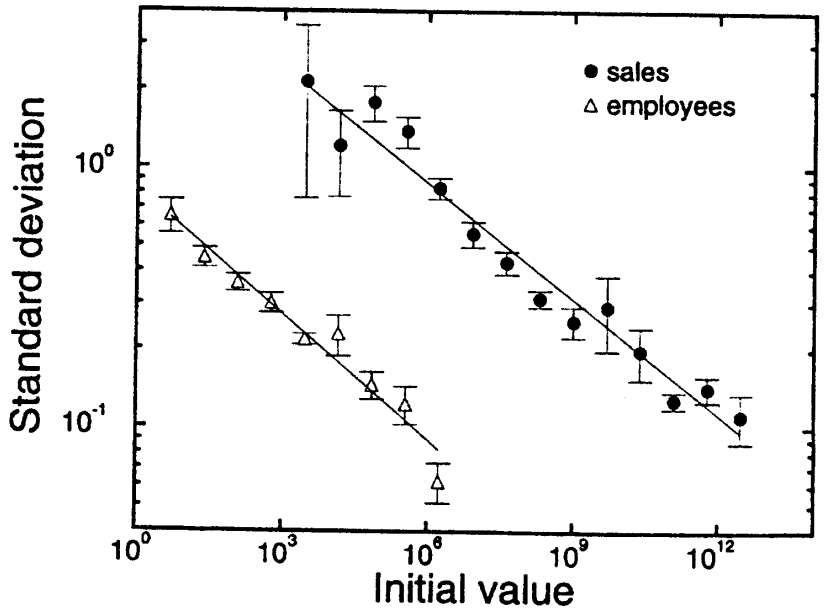
Firm Facts: Growth Rates are Laplace Distributed



Stanley, Amaral, Buldyrev, Havlin, Leschhorn, Maass,, Salinger and Stanley, *Nature*, *379* (1996): 804–6

$$r_t \equiv \ln \frac{S_{t+1}}{S_t} \qquad p(r) = \frac{1}{\sqrt{2}\sigma} \exp\left(-\frac{\sqrt{2}|r - \bar{r}|}{\sigma} \right)$$

More Firm Facts: Variance in Growth Rates Decreases with Firm Size



$S \sim r_0^{-\beta}$

$\beta \approx 0.15 \pm 0.03$ (sales)

$\beta \approx 0.16 \pm 0.03$ (employees)

Stanley, Amaral, Buldyrev, Havlin, Leschhorn, Maass,, Salinger and Stanley, *Nature*, *379* (1996): 804–6

Further Firm Facts

- **Wage rates are increasing in firm size:**
  - **Log(wages) α Log(size)**
- **More variance in job destruction time series than in job creation**
- **'Stylized' facts:**
  - **Growth rate variance falls with age**
  - **Probability of exit falls with age**

More Firm Facts

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- **Today there is no *microeconomic* explanation for these observations!**

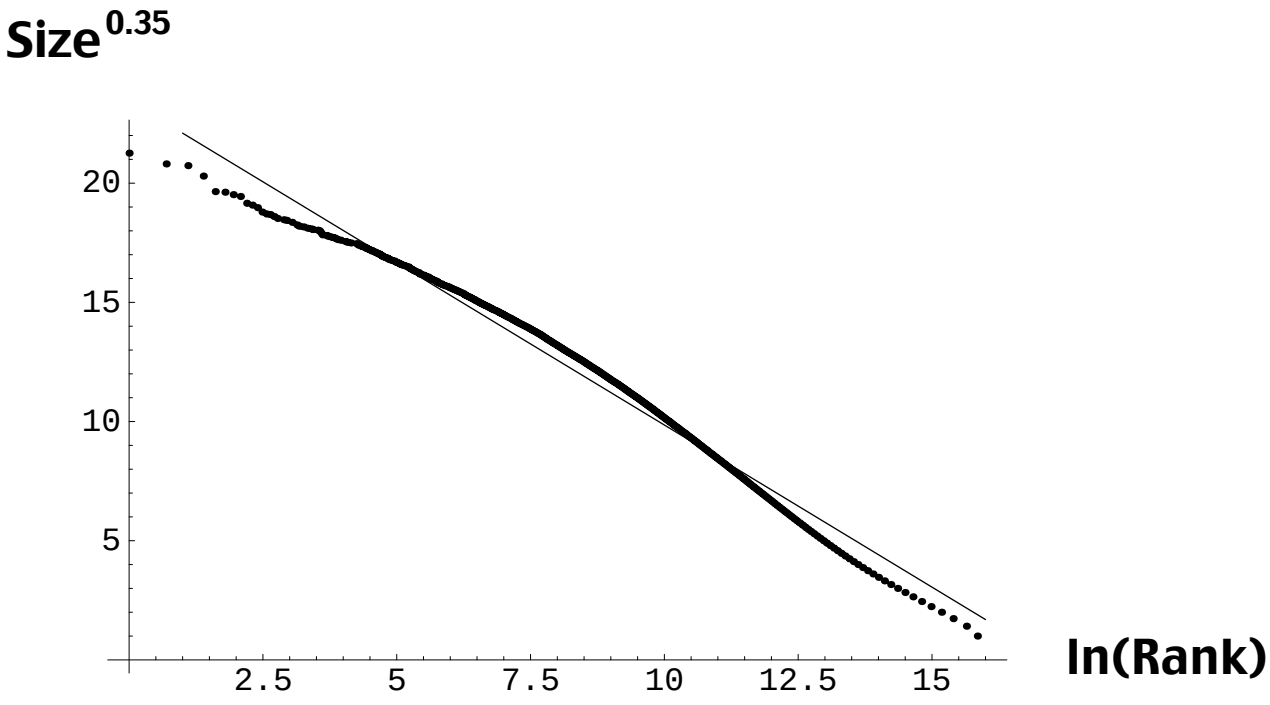
Requirements of an Empirically Accurate 'Theory of the Firm' (after Stanley *et al.* [1996])

- Produces a right-skewed (e.g., power law, log normal) distribution of firm sizes
- Generates Laplace (double exponential) distribution of growth rates
- Yields variance in growth rates that decreases with size according to a power law
- Wage-size effect obtains
- Methodologically individualist (i.e., written at the agent level)

Synopsis of Endogenous Firm Formation Model

- Heterogeneous population of agents
- Situated in an environment of increasing returns (team production)
- Agents are boundedly rational (locally purposive not hyper-rational)
- Rules for dividing team output (compensation systems)
- Agents have social networks from which they learn about job opportunities

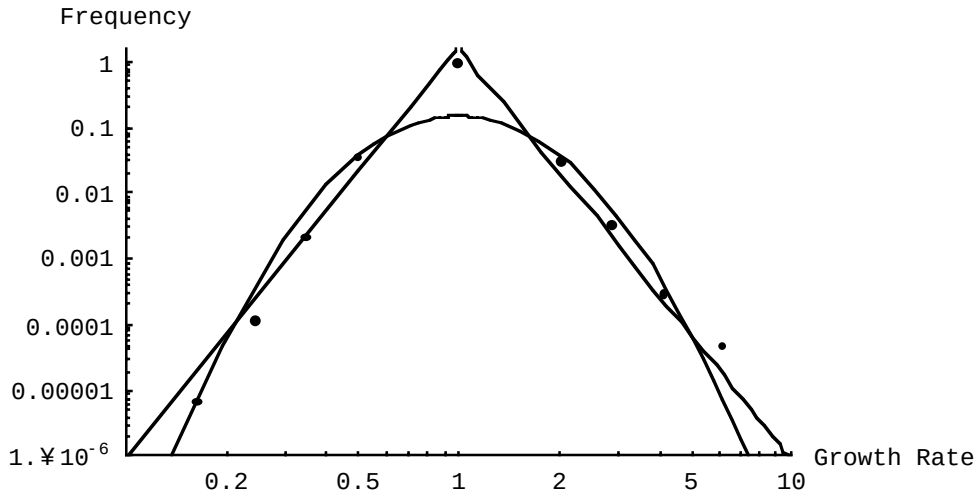
Firm Size Distribution from Model is Weibull Distributed



<u>Stochastic process models</u>: Gibrat process leads to log normal, Simon's model yields to Yule distribution (discrete Pareto)

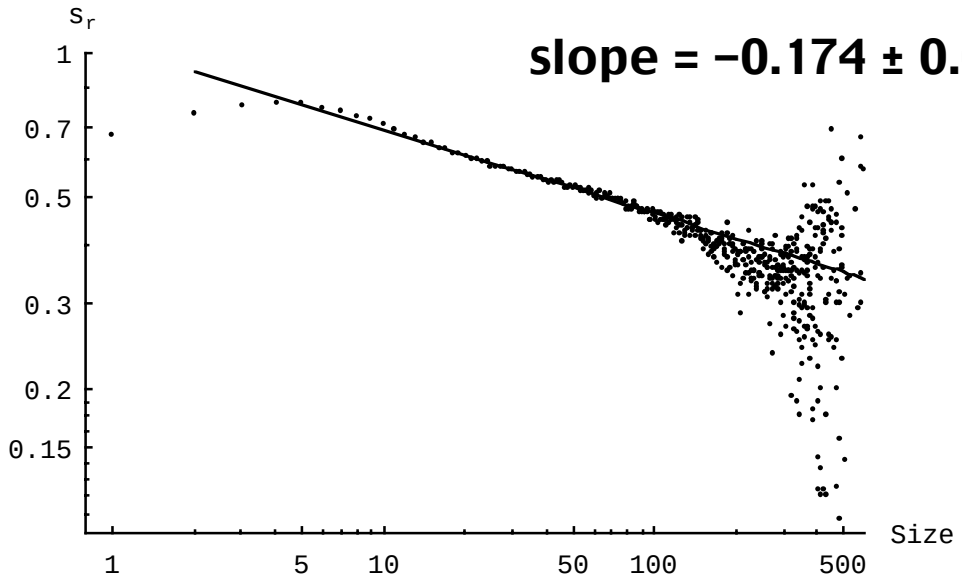
Firm Growth Rate Distribution

Growth rates Laplace distributed by K–S test



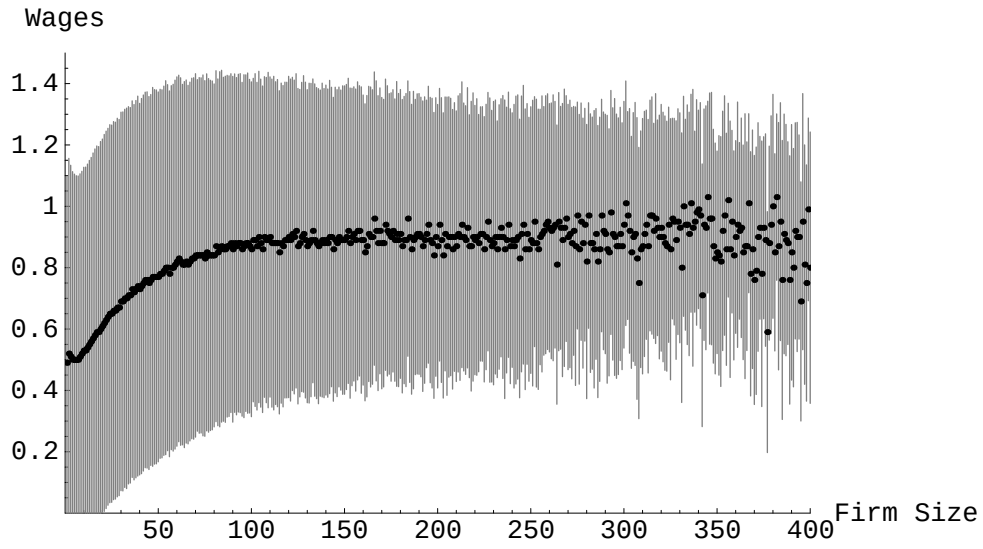
Stanley *et al* [1996]: Growth rates Laplace distributed

Variance in Growth Rates as a Function of Firm Size



Stanley *et al.* [1996]: Slope ≈ −0.16 ± 0.03 (dubbed 1/6 law)

Wages as a Function of Firm Size: Search Networks Based on Firms



Brown and Medoff[1992]: wages α size $^{0.10}$

Competing Theories of City Formation

- Positive–negative externality trade–off models (increasing returns vs. congestion)
  - E.g., A Marshall, J Jacobs, V Henderson
- 'Central Place' theory (Christaller [1933])
  - Recent formalization by Fujita, Krugman and Mori
- 'Nihilistic' stochastic process models
  - For example, Simon [1955], Hill [1974], Gabaix [1999]
  - Empirical orientation--these explain Zipf's law

Herbert Simon's Model

- There is some initial distribution of cities
- With probability $\varepsilon \ll 1$, a new city is born
- With probability $p = 1 - \varepsilon$, a population lump is added to an existing city in proportion to the city size (i.e., growth α size; growth *rate* independent of size)
- Yields a power law of city size as a function of rank with exponent $1 + \varepsilon$
- Similar model due to Steindl

Problems with these approaches

- Stochastic models *explain* the data but not 'economically,' i.e., they have little economic content
- Models with microeconomic content don't explain the data
- The riddle (Krugman *et al*. [1999]): "...at this point nobody has come up with a plausible story about the process that generates the rank–size rule..."

Story Behind the Model

- **People – Firms – Cities:**
  - People live in locations
  - People come together to form Firms
  - People migrate to better job opportunities
  - Local agglomerations of Firms are Cities
  - Productive Cities attract more People
  - Larger Cities foster more Firms
- **Human Capital Theory:**
  - Human capital driven growth (Jane Jacobs externalities, Lucas, Roemer, etc.)
- **Generates a stable system of cities or urban hierarchy**

Methodology

≠local purposiveness;
≠team (joint) production;
≠heterogeneous preferences/human capital;
≠adaptive individuals
≠constantly adjusting input
≠periodically jumping firms
≠ability to start-up new firms

≠increasing returns to human capital;
≠dynamic processes of firm formation and evolution;
≠finite firm lifetimes;
≠skewed size distribution;
≠successful firms attract human capital
≠firms are emergent

≠cities attract firms
≠big cities attract successful firms
≠path-dependent histories
≠movement up and down size distribution
≠occasional birth of new cities
≠cities are 'super-emergent'

higher levels of organization

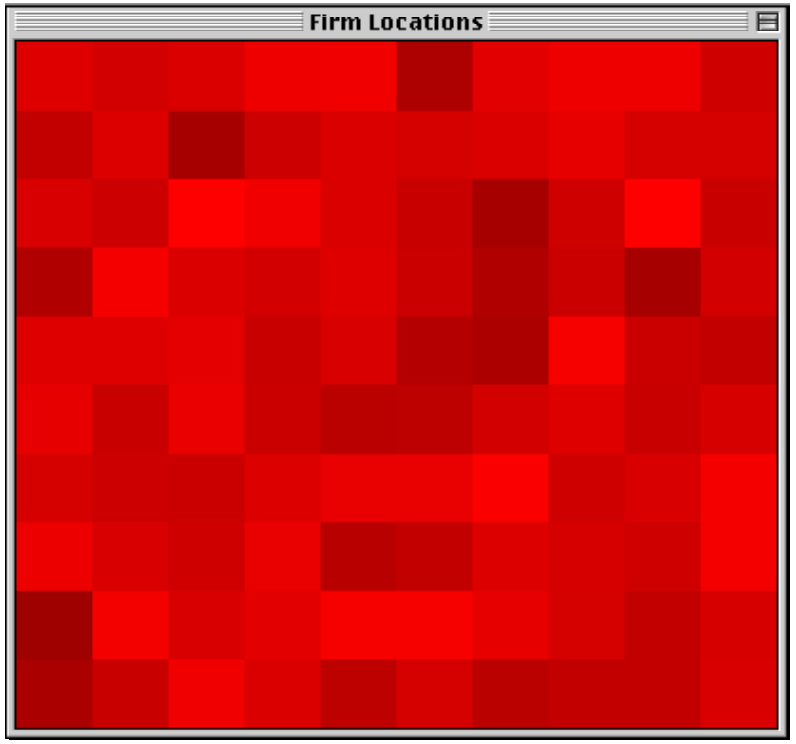
increasing complexity

City Formation Model

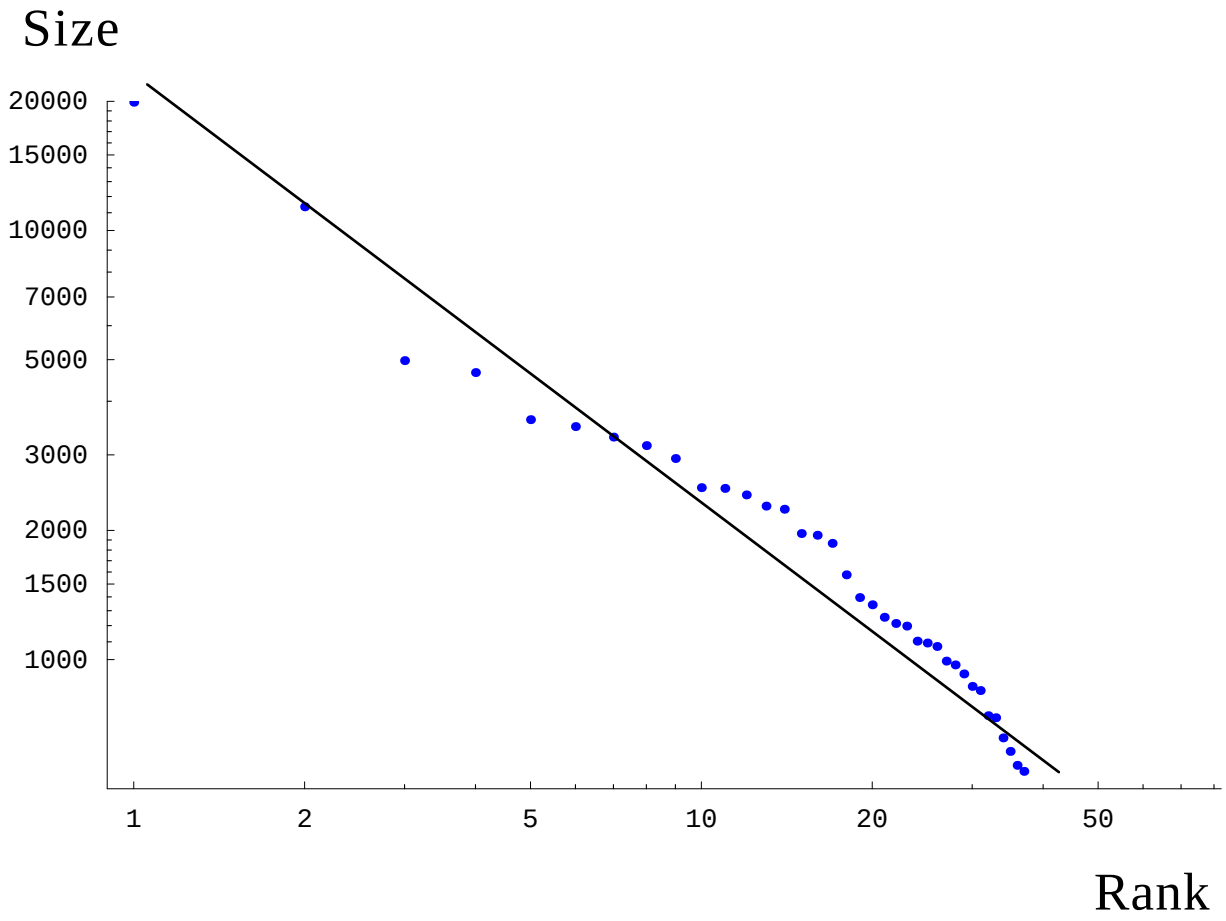
- There is a finite set of 'locations,' $L = \{a, b, c, \ldots, z\}$
- Each agent's initial location is random
- When an agent joins a firm it adopts the the firm's location (initial location of the founder)
- When an agent starts up a new firm:
  - with probability $\delta \ll 1$ it selects a random location
  - with probability $1 - \delta$ it keeps its present location

Typical Realization

- 10,000 agents
- Basic firms model:
  - increasing returns, α =2
  - uniformly distributed preferences
  - equal sharing
  - agents start as singletons
- Basic city model:
  - 100 locations
  - δ = 1/2 %
  - initial distribution of agents across locations is uniform



Model Yields Zipf's Law



Summary, I

An empirically-accurate theory of the firm:

- ✓ Produces a right-skewed (Weibull) distribution of firm sizes
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- ✓ Methodologically individualist (i.e., written at the agent level)

Summary, II

- Local increasing returns with free agent entry and exit is *sufficient to generate* firms and cities
- Highly non-stationary (turbulent) micro-data, stationary macro-data
- Constant returns at the aggregate level
- A microeconomic explanation of the empirical data
- Successful firms and cities are those that can attract and maintain high productivity workers
- Analytically difficult model tractable with computational agents

Future Work

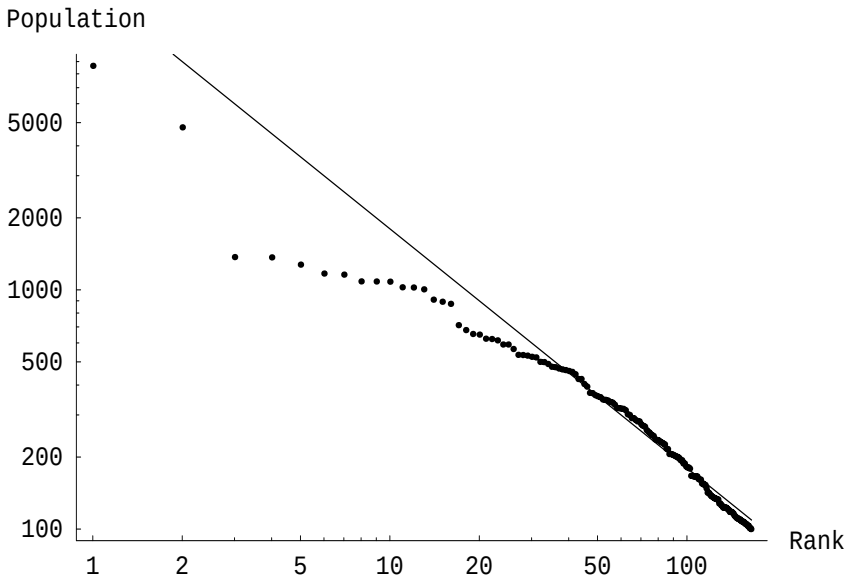
- Get internal structure to self-organize
  - Evolve governance structure within firms
  - Fractal dimension of intra-city geography

Future Work

- Get internal structure to self-organize
  - Evolve governance structure within firms
  - Fractal dimension of intra-city geography
- Compute stationary distributions analytically:
  - Sizes
  - Growth rates
  - Dependence of growth rate variance on size
  - Dependence of wages on size

Russia: Systematic deviation from Zipf

- 67 million people in largest 164 cities
- City size distribution is far from Zipf
- Too few large cities
- Insufficient human capital formation?
- We can compute amount of migration necessary
- Can we compute time needed for adjustment?



Five Speculations...

1. Microeconomic equilibrium theories will never explain firm and city size data

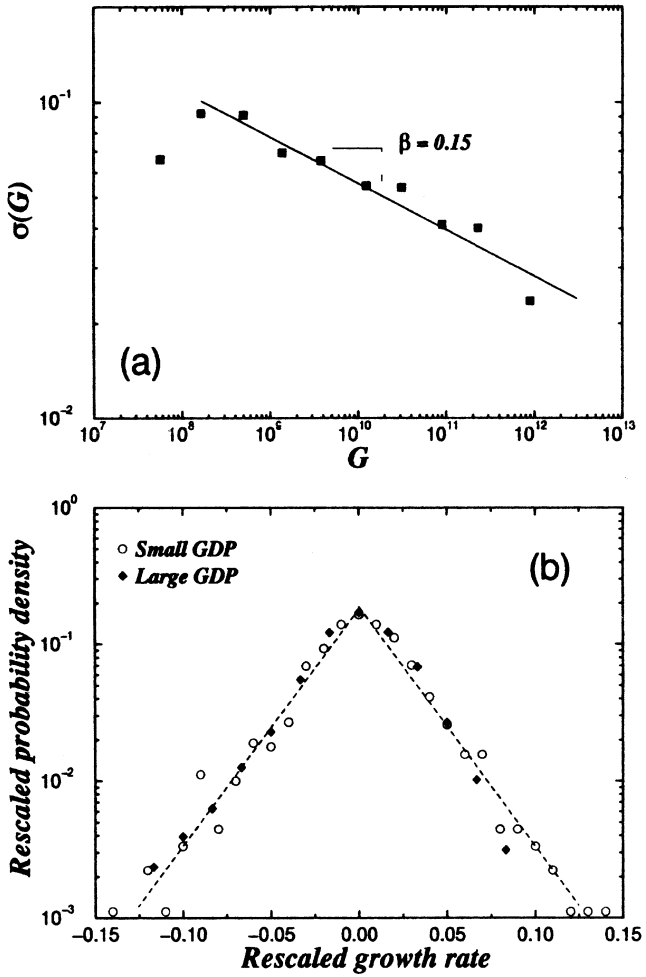
2. Many stationary aggregate data do not have explanations involving agent-level equilibrium

3. The focus of the conventional theory of the firm is highly *normative*

4. Cities are just agglomerations of firms

5. Countries too...?

Firms and Countries: Same Distribution of Growth Rates!



Canning *et al.*, *Economics Letters* (1998)