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interlocking explanations of cause and effect between disciplines

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More Power to You

Power laws and what they mean for investors

In the last few years the concept of self-organizing systems*–of complex systems in which randomness and chaos seem spontaneously to evolve into unexpected order–has become an increasingly influential idea that links together researchers in many fields, from artificial intelligence to chemistry, from evolution to geology. For whatever reason, however, this movement has so far largely passed economic theory by. It is time to see how the new ideas can usefully be applied to that immensely complex, but indisputably self-organizing system we call* the economy.

Paul Krugman
The Self-Organizing Economy[1]

Zipf It

Here's an activity to offset ennui on a rainy afternoon. Take a text—say, James Joyce's *Ulysses*—and for all the words plot the rank (from the most widely used words to the least-used) and frequency (how often each word occurs).[2] If you express this word distribution on a proportional log scale, you will find a straight line from the upper left hand of the chart to the bottom right-hand chart.[3]

George K. Zipf, a Harvard linguist, noticed this relationship in a number of systems in the 1930s and summarized them in his famous book *Human Behavior and the Principle of Least Effort*. Zipf's Law, as scientists came to call it, is actually only one example among many of a "power law." A power law implies that you see a few occurrences very frequently and large instances rarely.

Zipf erroneously argued that his law distinguished the social sciences from the physical sciences. Since his work, scientists have discovered power laws in many areas, including physical and biological systems. For example, scientists use power laws to explain relationships between the mass and metabolic rates of animals, frequency and magnitude of earthquakes (Gutenberg-Richter law), and frequency and size of avalanches. Power laws are also very prominent in social systems, including income distribution (Pareto's law), city size, Internet traffic, company size, and changes in stock price. Many people recognize power laws through the more colloquial "80/20 rule."[4]

Why should investors care about power laws? First, the existence of power law distributions can help reorient our understanding of risk. Most of finance theory—including models of risk—is based on the idea of normal or lognormal distributions of stock price changes. A power law distribution suggests periodic, albeit infrequent, price movements that are much larger than the theory predicts. This fat-tail phenomenon is important for portfolio construction and leverage.

Second, the existence of power laws suggests some underlying order in self-organizing systems. Even though scientists haven't fully explained the mechanisms that lead to power laws in social systems, we have enough evidence that power laws *exist* to make some structural predictions about what certain systems will look like in the future.

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Finally, standard economic theory does not easily explain these power laws. For example, neoclassical economics focuses on equilibrium outcomes and assumes that individuals are fully informed, rational, and that they interact with one another indirectly (through markets). In the real world, people are adaptive, are not fully informed, and deal directly with one another. So ideally we should seek to explain the empirical findings with an approach that fits how people really act.[5]

The More Things Change . . .

Zipf specified a very simple equation to express his law:

Rank x Size = Constant

This equation says that the quantity under study is inversely proportional to the rank. Given Zipf's equation, we can obtain a sequence by multiplying the constant by 1, ½, ⅓, ¼, etc. Take the case of city size distributions in Spain. If the largest city, Madrid, has 3 million inhabitants, the second-largest city, Barcelona, has one-half as many, the third-largest city, Valencia, one-third as many, and so forth. Zipf's Law does describe some systems well, but is too narrow to describe the variety of systems that exhibit power laws.

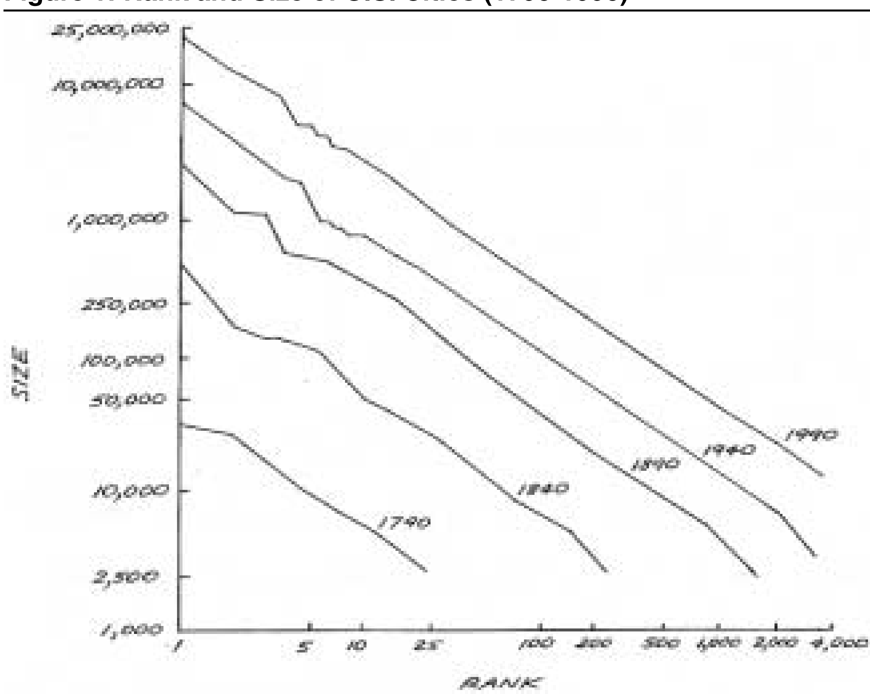
The brilliant polymath Benoit Mandelbrot showed that two modifications to Zipf's Law make it possible to obtain a more general power law.[6] The first modification is to add a constant to the rank. This changes the sequence to 1/(1+constant), 1/(2+constant), 1/(3+constant), etc.

The second modification is to add a constant to the power of 1 in the denominator. This yields $1/(1+\text{constant})^{1+\text{constant}}$, $1/(2+\text{constant})^{1+\text{constant}}$, etc. The modified power can be a whole number or an intermediate value (e.g., $1/(1+\text{constant})^{3/4}$). Zipf's Law is the special case where both constants are set to zero.

Even with the introduction of these two parameters, the generalization from Zipf's Law to a broader set of power laws remains very simple. That such an elementary equation describes such diverse phenomena certainly evokes wonder, especially since we have no unified explanation for how these power laws come about.

One of the interesting features of power laws in social systems is their robustness. For example, Figure 1 shows the plot for the rank and size in U.S. cities from 1790 to 1990. Notwithstanding population growth and substantial geographic shifts, the relationship between rank and size remained remarkably consistent for 200 years.

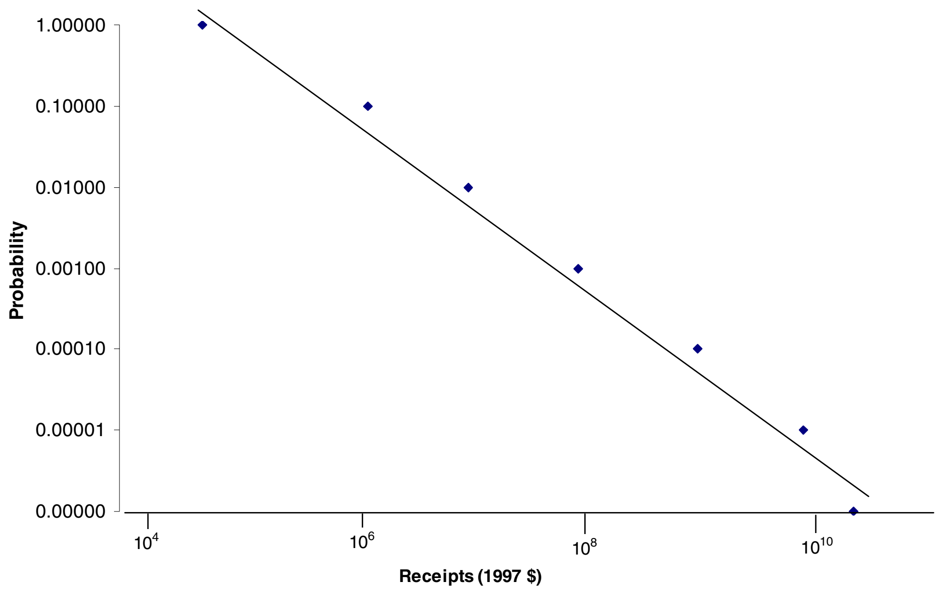
Figure 1: Rank and Size of U.S. Cities (1790-1990)



Source: David F. Batten, *Discovering Artificial Economics: How Agents Learn and Economies Evolve*, 165.

Another example, and more directly applicable for investors, is company size. Figure 2 shows that the relationship between sales and frequency for U.S. companies in 1997 follows Zipf's Law. Economist Rob Axtell created this chart based on U.S. Census Bureau data, which were not made available until early 2001, based on 5.5 million firms and over 100 million employees.

Figure 2: Sales and Cumulative Probability for U.S. Firms (1997)



Source: Robert L. Axtell, "Zipf Distributions of U.S. Firm Sizes," *Science,* September 7, 2001, 1819.

Axtell notes that the distribution of firm sizes is insensitive to changes in political and regulatory environments, mergers and acquisitions waves, new firm and bankruptcy trends, and even large-scale demographic transitions within the workforce (e.g., women entering the U.S. workforce).[7] The implication is that there are important underlying mechanisms that create the order we see.

No one really completely understands the mechanisms that yield power laws, but there are a number of models or processes that generate them.[8] Perhaps the best known is "self-organized criticality"—a model popularized by theoretical physicist Per Bak. Bak suggests a scene where a child is at a beach letting sand trickle down into a pile. At first the pile is relatively flat and the grains remain close to where they fall. Once the pile becomes steeper, additional grains will periodically trigger a little sand slide. A while longer and the sand slides will be as big as the pile itself. The system is in a "critical" state—between steady state and randomness. Once the pile is in a critical state, additional grains produce sand slides of varying magnitudes and the sizes of the sand slides follow a power law.[9]

There are aspects of the sand pile metaphor that are useful for thinking about social systems. For one, the economic systems are clearly self-organizing. That is, most companies, cities, or countries that we see are the result of interaction between individuals, not of central planning. Also, there is a sense of a "critical" state. In a physical system, a critical point is one where just a small change produces a phase transition—for example, water freezes as the temperature drops below zero degrees centigrade. Economists do not define critical points as clearly for economic systems, but we do know that individuals neither stay at the same company forever (steady state) nor haphazardly jump from company to company (randomness). Axtell has captured these features through an agent-based model to explain firm and city sizes. His model yields results that are consistent with the empirical data.[10]

Catch the Power

There are a number of ways that an understanding of power laws helps investors. The first way builds off Axtell's work on company size. Given the evidence that power law distributions are robust over time, we have a good sense of *what* the distribution will look like in the future even though we have no idea *where* individual companies will fall within it. But given reasonable assumptions for economic growth and inflation, we can derive a good estimate of the probabilities of companies being of a particular size.

We know ahead of time, for example, that a miniscule percentage of companies will be very large (e.g., >$200 billion sales). We can look at the imputed growth rates of large companies today and discern how many of them are projected, based on expected growth, to be very large. If the group *projected* to be very large vastly exceeds the percentage that *will* be large, we know there is the likelihood of substantial downward expectation revision.

Another way investors can use power laws is to understand the topology of the Internet. A classic example of a self-organizing network, the Internet has spawned a host of power law relationships—including the number of links per site, the number of pages per site, and the popularity of sites. These power laws suggest uneven benefits for companies that make heavy use of the Web.[11] The development of the Web may be instructive for the organization of future networks.

Power laws represent a number of social, biological, and physical systems with fascinating accuracy. Further, many of the areas where power laws exist intersect directly with the interests of investors. An appreciation of power laws may provide astute investors with a useful differential insight into the investment process.

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[1] Paul Krugman, *The Self-Organizing Economy* (Oxford: Blackwell Publishers, 1996), vi.

[2] George Kingsley Zipf, *National Unity and Disunity: The Nation As a Bio-Social Organism* (Bloomington, Indiana: Principia Press, 1941), 398-399.

[3] For example, in log 10 the scale would be 10^1 (=10), 10^2 (=100), and 10^3 (=1,000) versus the more familiar 10,11,12.

[4] Richard Koch, *The 80/20 Principle: The Secret to Success by Achieving More with Less* (New York: Currency, 1998).

[5] Rob Axtell, "Zipf's Law of City Sizes: A Microeconomic Explanation *Far from Equilibrium*" presentation at a RAND workshop, *Complex Systems and Policy Analysis: New Tools for a New Millennium*, held September 27 and 28, 2000.

[6] These modifications are lucidly explained in Murray Gell-Mann, *The Quark and the Jaguar: Adventures in the Simple and the Complex* (New York: W.H. Freeman, 1994), 92-100.

[7] Robert L. Axtell, "Zipf Distribution of U.S. Firm Sizes," *Science, vol. 293*, September 7, 2001, 1818-1820. See http://www.sciencemag.org/content/vol293/issue5536/index.shtml

[8] These include self-organized criticality, highly optimized tolerance (HOT); and the Gibrat process. Not all of these processes are mutually exclusive.

[9] Per Bak, *How Nature Works: The Science of Self-Organized Criticality* (New York: Springer-Verlag, 1996), 1-3.

[10] Robert Axtell, "The Emergence of Firms in a Population of Agents: Local Increasing Returns, Unstable Nash Equilibria, and Power Law Size Distributions," *Brookings Institution, Center on Social and Economics Working Paper no. 3*, June 1999. Also, Robert L. Axtell and Richard Florida, "Emergent Cities: A Microeconomic Explanation of Zipf's Law," *Brookings Institution and Carnegie Mellon University Working Paper*, September 2000.

[11] Albert-László Barabási, *Linked: The New Science of Networks* (Cambridge, MA: Perseus, 2002), 69-72; Bernardo A. Huberman, *The Laws of the Web: Patterns in the Ecology of Information* (Cambridge, MA: MIT Press, 2001), 25-31; Lada A. Adamic, "Zipf, Power-laws, and Pareto – a Ranking Tutorial," *Xerox PARC Working Paper*. http://ginger.hpl.hp.com/shl/papers/ranking/ranking.html

Other resources:

A great Web site for all things Zipf: http://linkage.rockefeller.edu/wli/zipf/

A fun site to see Zipf in action around the world: http://www.oup.co.uk/best.textbooks/economics/marrewijk/zipf/

Finance-related work: Ilija I. Zovko and J. Doyne Farmer, "The Power of Patience: A Behavioral Regularity in Limit Order Placement", *Santa Fe Institute Working Paper # 02-06-027*, June, 2002. www.santafe.edu/sfi/publications/wpabstract/200206027

James H. Brown and Geoffrey B. West (eds.), *Scaling in Biology* (Oxford: Oxford University Press, 2000).

Yuji Ijiri and Herbert A. Simon, *Skew Distributions and the Sizes of Firms* (New York: North-Holland, 1977).

Benoit B. Mandelbrot, *Fractals and Scaling in Finance: Discontinuity, Concentration, Risk* (New York: Springer, 1997).

Manfred Schroeder, *Fractals, Chaos, and Power Laws: Minutes from an Infinite Paradise* (New York: W.H. Freeman, 1991).

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