Scaling and universality in the micro-structure of urban space

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Abstract

We present a broad, phenomenological picture of the distribution of the length of urban linear segments, l, derived from maps of 36 cities in 14 different countries. By scaling the Zipf plot of l versus its rank, two curves are obtained which are dependent on city location (geography) but not on city size. It is shown that a third class of cities does not fall into the above universality classes. This collapse of curves suggests that urban morphology is governed by similar statistical rules at a geographical scale.

Key words: Fractals, Urban planning, Scaling laws, Universality.
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1 Introduction

The morphology of urban settlements and its dynamics has captured the interest of physicists [1–9] as it may shed light on Zipf's law for cities [4,5,10,11], challenge theoretical frameworks for cluster dynamics or improve predictions of future urban growth [2,6,7].

The search for a 'unified' theory of urban morphology has focused on the premise that cities can be conceptualized at several scales as fractals. At the

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regional scale, rank-order plots of city size follow a fractal distribution [1] and population scales with city area as a power-law [12]. More recently, it has been observed that the area distribution of satellite cities, towns and villages around huge urban centers also obeys a power-law with exponent ≈ 2 [2,6]. At the scale of transportation networks, railway networks appear to have a fractal structure [13]. At the scale of the neighbourhood, it has been suggested that urban space resembles a Sierpinsky gasket [1,12]. These scales are inter-related as summed up in [1] (author's translation from French): 'The polycentric growth, which is connected to the non-homogeneous distribution of pre-urban cores and the birth of a hierarchy of sub-centres, influences the morphology of the transport network, which plays in itself an important role for the axial growth and therefore for the future spatial development of the urbanised area'.

Urban fractal growth is essentially a space filling process, that is, the larger the fractal dimension value, the more filled a planar city becomes. The fractal dimensions of US cities and international cities were obtained with values ranging from 1.2778 (Omha, [14]) to 1.93 (Beijing, [1]), where fractal dimension of large cities tend to cluster around the latter value [1,12,15,14]. The fractal dimensions representing urban growth of London between 1820 − 1962 were determined with values ranging from 1.322 to 1.791 [12]. The fractal dimensions for the growth of Berlin in 1875, 1920 and 1945 were also reported to be 1.43, 1.54 and 1.69, respectively [1]. The fractal dimension of urban aggregates is a global measure of areal coverage. Nevertheless, it does not show whether clusters are separated or connected [7]. Equally, the fractal dimension does not yet fully characterise the microstructure of urban space on the giant cluster that grows around the city core (or 'central business district'), as the available datasets do not have, in general, enough resolution (e.g., for US Bureau of Census data [16] each pixel area in the map represents an area of $178m \times 178m$ on the ground [14]) —nevertheless, the situation is quite different for studies of individually selected cities, where resolution can be quite high [17]. In other words, detailed measures of both areal coverage and spatial distribution are needed to complement the description of the morphology of an urban area adequately.

Hillier and Hanson [18] suggest an underlying structure to urban space that is determined by the complexity of buildings which bound the space [19]. Urban space available for pedestrian movement, excluding by definition physical obstacles, is relatively linear. When human beings are walking along this free space, such a free space is locally perceived as a 'vista' which can be approximately represented as a line. The global set of vistas, so-called axial map, is defined as the least number of longest straight lines. An axial map can be derived by drawing the longest possible straight line, then the next longest line, so-called axial line, until the free space is crossed; and finally "all axial lines that can be linked to other axial lines without repetition are so linked"

[18,20].

Here we show that we can rescale axial line length and rank to obtain two distinct rank-order curves that provide a semi-geographical classification for several cities independently of city size. We also show that there is a class of cities that do not obey this classification. The collapse of curves suggests that spatial fluctuations in the length of urban linear structures, differing in size and location, are governed by similar statistical rules and supports the hypothesis that the linear dimension of large scale structures in cities reflects generic properties of city growth [21].

2 Structure of urban space

Let $l_i = \{l_{i,j}\}$, $j = 1, \cdots, N_i$, be the N_i axial lines associated with city i. Each axial line, $l_{i,j}$ ($j = 1, \cdots, N_i$) is defined by the coordinates of its extremities

$$l_{i,j} = \left\{ \left(x_{(i,j),1}, y_{(i,j),1} \right), \left(x_{(i,j),1}, y_{(i,j),2} \right) \right\}$$

The axial map of city i, C_i, is thus a set of N_i points on a fourth dimensional space, $C_i = \left\{ \left(\rho_{i,j}, \theta_{i,j}, l_{i,j}, \varphi_{i,j} \right) \right\}$, where (ρ_i, θ_i) are the polar coordinates of the axial line geometric centre, $s_{i,j} = \left(\frac{x_{(i,j),1}+x_{(i,j),2}}{2}, \frac{y_{(i,j),1}+y_{(i,j),2}}{2} \right)$, and $\left(\frac{l_{i,j}}{2}, \varphi_{i,j} \right)$ the polar coordinates of the axial line's extremities on the geometric centre reference system, $d_{i,j} = \left(\frac{\left|x_{(i,j),1} - x_{(i,j),2}\right|}{2}, \frac{\left|y_{(i,j),1} - y_{(i,j),2}\right|}{2} \right)$. Figure 1 shows the axial map of Tokyo, where lines longer than 1000 m are traced in black.

Coordinates ρ and θ encode the geographic location of axial lines. The unconditional distribution of φ is multimodal for rather general families of urban settlements. This occurs, for example, in the case where land is partitioned in clusters of randomly oriented orthogonal grids. Nevertheless, the unconditional distribution of l is unimodal and skewed to the right (see Figure 2), and, thus, a good candidate for inspection of intermittency in urban space. We fit the data to a stretched exponential distribution [22, pp 153-154] in Figure 2 (a), but verify that the fit is unsuitable to describe the large events.

It has been noted that in any finite critical system, power-law descriptions must give way to another regime dominated by finite size effects and the distributions in general cross over to an exponential decay, which leads to a curvature in the log-log plots [23]. However, spatial auto-correlations are evident from Figure 1 for the large events, questioning the validity of a rigorous fit of the distribution tail to a univariate pdf. Nevertheless, as we shall see, auto-correlations do not seem to affect the overall shape of the distribution.

We analyse the unconditional probability distribution of (axial) line length of

36 cities in 14 di¤erent countries (see Table 1). In our analysis we use the rank-order technique [22]. In Figure 3(a) we shift the y axis to show the curves. To interpret the apparently unsystematic data in Figure 3(a) e¤ectively, it is instructive to scale the data [24]. Since the rank ranges between 1 and max$(rank_i)$, we de…ne a scaled relative rank $rank_i/\max(rank_i)$. Similarly, for the ordinate, it is useful to de…ne a scaled line length by $l_i/\langle l_i \rangle$. As shown in Figure 3(b), there is relatively good collapse of the data sets onto two master curves for 25 (the cities plot in red and blue) of the 36 cities under study. The other 11 cities (plot in green in Figure 3) do not collapse clearly onto a single curve (see inset of Figure 3(b)).

3 Discussion

Our results are unexpected in that two universality classes appear for a wide range of cities.

Do we con…rm the hypothesis of Batty&Frankhauser that urban space is like a Sierpinski gasket?

It has been proposed that the evolution of urban aggregates in time approaches a Pareto distribution [25], which suggests that spatially developed urban aggregations exhibit a 'quasistationary' state. As the Pareto exponents of line length of di¤erent urban settlements are similar, the Pareto exponent could serve, in the scope of certain error limits, as a structural measure of developed urban aggregates, which may complement the fractal dimension.

Empirical explorations of urban morphology may lead to unexpected geometrical layouts where power-laws are present and which are challenging for the physicist.

The possibility that line length has a scale-free structure indicates that strategic targeting of planning of long streets creates the main …ngerprint of a city.

Our …ndings show that it is important to model urban aggregates in detail , and that it may be more useful to model urban morphology as random rather than as the outcome of rational decisions for some purposes.

Navigating in our cities is a complex perceptual task that may be added by a fractal morphology: if there is no characteristic scale, then the navigator is familiar with his surroundings at every scale -one decision mechanism adds him in navigating at several di¤erent scales.

Comment that line length appears to be self-similar accross morphologically relevant ranges of scales (2-3 orders of magnitude). If so, their characteristics can be described with fractal geometry, that is, simple scaling laws

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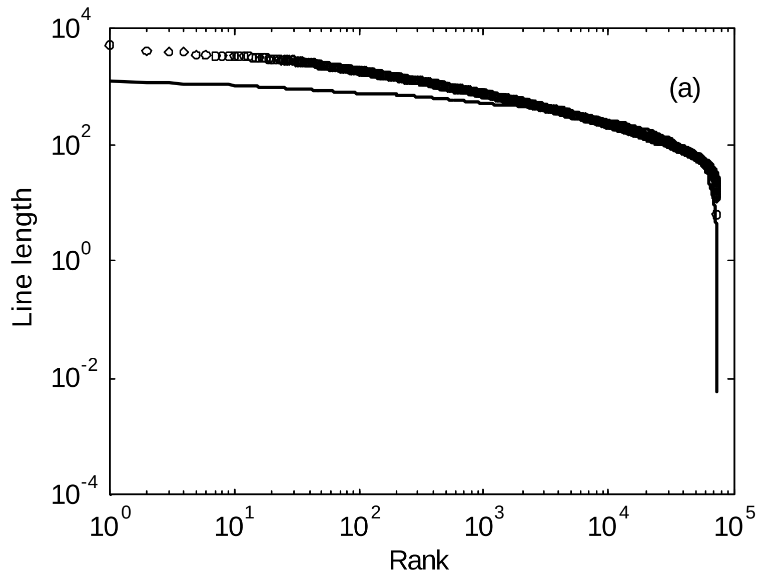
Fig. 1. Axial map of Tokyo. The 42310 lines with length less than 100 m are omitted. The 30927 lines with $10^2 < l < 10^3$ (m) are in grey and the 525 lines with $l > 10^3$ (m) in black.



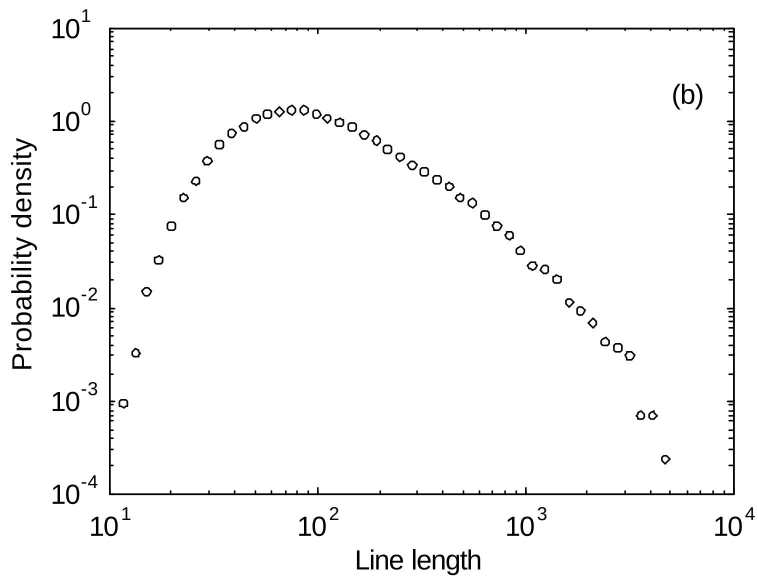


Fig. 2. Data are shown for the city of Tokyo. (a) Rank-order plot of line length (circle points) together with a …t of the data to a stretched exponential pdf (solid line). (b) Unconditional probability density of line length.

Table 1
Geographical location and number of lines of the cities analysed.

Country	City	Number of lines
Japan	Tokyo	73753
U.S.A.	Chicago	30571
Chile	Santiago	26821
Thailand	Bangkok	24223
Greece	Athens	23329
Turkey	Istanbul	21798
U.S.A.	Seattle	20213
U.K.	London	15969
U.S.A.	Baltimore	11636
Netherlands	Amsterdam	9619
U.K.	Bristol	7028
U.S.A.	Las Vegas	6909
Iran	Shiraz	6258
Cyprus	Nicosia	6023
Netherlands	Eindhoven	5782
U.K.	Milton Keynes	5581
Spain	Barcelona	5575
U.K.	Wolverhampton	5423
India	Ahmenabad	4876
U.S.A.	New Orleans	4846
Iran	Kerman	4372
U.K.	Nottingham	4365
U.K.	Manchester	4308
U.S.A.	Pensacola	4296
Iran	Hamadan	3855
Iran	Qazvin	3723
Netherlands	The Hague	3350
U.K.	Norwich	2119
U.S.A.	Denver	2092
Iran	Kermanshah	1870
U.K.	York	1773
Iran	Semnan	1770
Bangladesh	Dhaka	1566
Hong Kong	Hong Kong	916
U.K.	Hereford	854
U.K.	Winchester	616

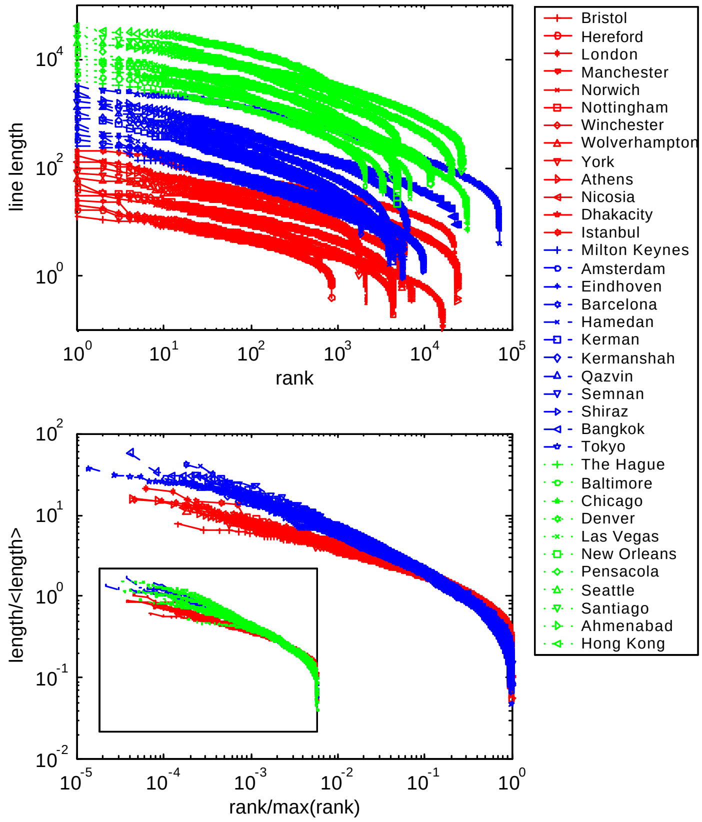


Fig. 3. (a) Rank-order plot of line length. Plots are shifted along the y axis for clarity. (b) Scaled rank-order plot of line length. Curves clearly collapse for two sets of cities, but not so clearly if a third set is introduced (inset).