1 Can physicists contribute to geography [1] ?

Statistical physics deals with systems comprising a very large number of interacting subunits, for which predicting the exact behaviour of the individual subunit would be impossible. Hence, one is limited to making statistical predictions regarding the collective behaviour of the subunits [1]. Recently, it has come to be appreciated that many such systems which consist of a large number of interacting subunits obey universal laws that are independent of the microscopic details. The finding, in physical systems, of universal properties that do not depend on the specific form of the interactions gives rise to the intriguing hypothesis that universal laws or results may also be present in economic and social systems [2,3] [2] .

If there is one message that emerges clearly from statistical physics, it is that sometimes *the details do not matter*. That, in a nutshell, is what is meant by universality. This is a way of saying that *collective* behaviour tends to be robust, and shared by many apparently different systems [4].

1.1 What is scaling? and universality?

Scaling may be expressed as a relatively compact statement:

$$p(\lambda l) = \lambda p(l)$$

A scale invariant system has the same statistical properties and hence "looks almost the same" at many different scales of observation. The actual object is different, but since its *statistical* properties are the same, one cannot readily distinguish the original complex object from a magnification of a part of it [5].

For a vivid analogy, recall the infamous Tacoma Narrows Bridge that once connected mainland Washington with the Olympic peninsula [3] . One day it

[1] This notes are based on extracts from papers written by physicists. Please consult references for the full text.

[2] An often-expressed concern regarding the application of physics methods to the social sciences is that physics laws are said to apply to systems with a very large number of subunits (of order of 10^{20}) while social systems comprise a much smaller number of elements. However, the "thermodynamic limit" is reached in practice for rather small systems. For example, in early computer simulations of gases or liquids reasonable results are obtained for systems with $20 - 30$ atoms.

[3] The bridge collapsed on November 7, 1940 at approximately 11:00 a.m. and had been open to traffic for only a few months. The reader is invited to view historical film footage which shows in 250 frames (10 s!) the

suddenly collapsed after developing a remarkably "ordered" sway in response to a strong wind. Students learn the explanation for this catastrophe: the bridge, like most objects, has a small number of characteristic vibration frequencies, and one day the wind was exactly the strength required to excite one of them. The bridge responded by vibrating at this characteristic frequency so strongly that it fractured the supports holding it together. The cure for this "diseased bridge" was a design that is capable of responding to many different vibration scales in an approximately equal fashion, instead of responding to one frequency excessively.

What about universality, the notion in statistical physics that many laws seem to be remarkably independent of details?

It was found empirically that one could form an analog of the Mendeleev table if one partitions all critical systems into "universality classes". It was found that quite disparate systems behave in a remarkably similar fashion near their respective critical points –simply because near their critical points what matters most is not the details of the microscopic interactions but rather the nature of the "paths along which order is propagated" [6].

Newcomers to the field of scaling invariance often ask why a power-law does not extend 'forever' as it would for a mathematical power-law of the form $f(x) = x^{-\alpha}$. This legitimate concern is put to rest by reflecting on the fact that power-laws for natural phenomena are not equalities, but rather are asymptotic relations of the form $f(x) \sim x^{-\alpha}$. Here the tilde denotes *asymptotic equality.* This means that $f(x)$ becomes increasingly like a power law as $x \to \infty$.

For a discussion on scaling and universality, the reader should refer to [3,5].

1.2 What are power-law distributions?

Frequency or probability distribution functions (pdf) that decay as a power-law of their argument

$$p(x)\,dx = p_0 x^{-(1+\alpha)} dx$$

have acquired a special status in the last decade and are sometimes called "fractal". A power-law distribution characterizes the absence of a characteristic size: *independently* of the value of x, the number of realizations larger than λx is $\lambda^{-\alpha}$ times the number of realizations larger than x. In contrast, an exponential for instance does not enjoy this self-similarity [4], as the existence

maximum torsional motion shortly before failure of this immense structure [http://cee.carleton.ca/Exhibits/Tacoma_Narrows/].

[4] An object is said to be self-similar if it looks "roughly" the same on any scale.

of a characteristic scale destroys this continuous scale invariance property [7]. In words, a power-law pdf is such that there is the same proportion of smaller and larger events, whatever the size one is looking at within the power-law range.

Power-law pdfs have the characteristic that the sample mean does not approach a limiting value as more data is collected (the averaged measures will either increase or decrease with the amount of data analyzed). There is no single value that we can identify as the "right" value for the average. Therefore, the population mean does not exist. For an insightful discussion on power-law pdfs see e.g. [8].

Physicists may care passionately if there are analogies between physics systems they understand (like critical point phenomena) and geographical systems they do not understand. But why should anyone else care? One reason is that scientific understanding of earthquakes moved ahead after it was recognized that extremely large events –previously regarded as statistical outliers requiring for their interpretation a theory quite distinct from the theories that explain everyday shocks– in fact possess the identical statistical properties as everyday events; e.g., all earthquakes fall on the same straight line on an appropriate log-log plot.

Finally, a current interesting hypothesis is that possibly one reason that diverse systems in such fields has physics, biology, and ecology have quantitative features in common may relate to the fact that the complex interactions characterizing these systems could be mapped onto some geometric system, so that scaling and universality features of other complex systems may ultimately be understood in terms of the connectivity of geometrical objects [5].

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