Optimal-Robust Design of Water Distribution Networks under Uncertain Scenario

Giustolisi O.[1], Mastrorilli M.[2]

[1] Civil and Environmental Engineering Department, Technical University of Bari, Engineering Faculty of Taranto, v.le del Turismo n.8, Taranto, 74100, Italy
[2] Environmental and Sustainable Development Engineering Department, Technical University of Bari, Engineering Faculty of Taranto, v.le del Turismo n.8, Taranto, 74100, Italy

Abstract

Looking at the Public Finance that must satisfy several social claims, nowadays investment efficiency for both an improvement and a better maintenance of the public assets has gained a central role for the Local Institutions and Communities. This paper is focalized on these issues, in particular its contribution is supposed to be in the improvement and innovation of the procedures in order to gain in efficiency of the water system. Care is taken about the urban water distribution systems, this is pursued by this project addressing the research towards the robust automatic design, under uncertainty of the water demand and of pipe consumption (i.e. variable roughness). This procedure is purposed to make water systems cheaper both under the starting investments' point of view and under the management point of view. A population-based optimisation strategy is used in order to face the aforementioned problems. On the one hand this population of solutions is randomly evolved; on the other hand the search is guided by one or more objective functions. The definition of the objective functions is a key issue too. Such techniques imply several advantages to the decision support, since they are able to solve highly complex problems, which involve a number of objectives and variables and therefore difficultly approachable by traditional techniques. The population-based techniques here introduced are the Genetic Algorithm. They were presented for the first time by Holland (1975) but their exploitation was encouraged by the wide diffusion of powerful and cheap computers and by the studies carried on by Goldberg (1989). The Multi-objective Genetic Algorithms here are integrated with Montecarlo sampling techniques based on the variance reduction, such as the "Latin Hypercube". These techniques allow an accurate sampling of those probability functions, which describe the uncertainty on the design conditions. The definition of these probability density functions related to the uncertainty and the optimisation of the algorithms' run times are among the main issues of the work. The innovation applied to the water distribution system design allows a general saving operational costs and investments as proved by a wide international literature.

Italian rules background

In Italy, the Act 36, 5/01/94, namely "Galli Act", supports the intelligent and sustainable use of the water, hinted by the Act 183, 18/05/89. The rationalisation of the water use is pursued by assigning the task of planning and control to an Institutional District Authority and the task of management of water resources, distribution, sewage and treatment to the water industry. The price of the service has to cover the water industry's operational costs, investments and financial measures related to the service, which has to satisfy the demands (effectiveness) claimed by the District Plan. The rules about the evaluation of the price are presented by the Appendix to the Institutional Decree 1/06/96, namely "Normalised methodology about the determination of the cost components and the evaluation of the price for the comprehensive water service". This Decree emphasizes the efficiency, effectiveness and economical advantageous management hinted by the "Galli Act". This methodology is addressed by the economical literature as "Price Cap". This has to encourage investment plans in an economically balanced scenario, in particular if its application is subjected to specific adjustments. The Price Cap contains one adjustment of these: the improvement of the service efficiency is encouraged in order to reduce the operational costs and then advantaging the investments (Section 6, first paragraph). The Price Cap explicitly addresses the strategic value of the investments in order to gain in efficiency. This approach is economically advantageous for the management able to efficiently run the company's investments, since these can return an operational efficiency which is higher than the improvement claimed by the District Plan. The Local Institutions and Communities are also advantaged by this investment efficiency, since this implies both an improvement and a better maintenance of the public assets.

The hydraulic problem and the optimal design by GA

The starting point for the scientific/technical problem of the optimal-robust design of water distribution systems is its mathematical formulation based on the least cost criterion [3,7]:

$$\begin{cases} A_{nodes}^{T} Q_{pipes} = Q_{nodes} \\ R_{pipes} Q_{Pipes} \left| Q_{pipes} \right| + A_{nodes} H_{nodes} = -A_{tanks} H_{tanks} \\ \sum_{i=1}^{pipes} C\left(d_{i,j}, L_{i,j}\right) = \min \\ h_i - h_{geodetic}^{i} - X \geq 0 \\ V_{i,j} - V_{max} \geq 0 \end{cases} \tag{1}$$

The first two equations from the mathematical system (1) are the matrix-form equations of the hydraulic system and they return the pipes' discharges (Q_{pipes}) and the nodes' hydraulic head (H_{nodes}) when (1) the function representing the water demand in nodes (Q_{nodes}), (2) the pipes' frictions (R_{pipes}) and (3) the water level in tanks/reservoirs ($\mathrm{H}_{\mathrm{tank}}$) are given. The influence matrix of nodes (A_{nodes}) represents the topology of the water distribution network. The remaining equations in the mathematical system (1) are design constraints. The first constraint is related to the economical cost of the network (they can be defined according to a more generic equation than in (1)) as function of the pipes' diameters ($d_{i,j}$) and length ($L_{i,j}$) . The second and the third constraints are concerned with the levels of service (minimum pressure in nodes X) and with the maximum velocity of water in pipes (V_{max}). The mathematical system (1) has pipes' diameters as decisional variables, which represent the target of the design task. When a set of diameters is given, there exists a set of R_{pipes} values, then of H_{nodes} and Q_{pipes}, returned by the solution of the first two equations of the mathematical system (1), and the economical cost of the network. The optimisation problem is particularly related to the constraints on the levels of service (minimum pressures in nodes).

Therefore, in a strict mathematical scenario, the problem of design a water distribution system is a non-linear optimisation problem, constrained to a combinatory space, since the diameters are discrete, and it has a very large number of local solutions.

A population-based optimisation techniques allows to efficiently approach such a problem, since they pursue a quite global exploration of the solutions' domain, which implies a high probability of getting the global optimum. Genetic Algorithms (GA) are population-based optimisation techniques. They are based on a simplified reproduction of the natural selection described by the evolutionary theories of Darwin [8,9]. GA mimic the biological evolution of the individuals and then they encourage the survival of the fittest despite the less fit individuals. In water distribution systems, the individuals are represented by sets of diameters assumed for each pipe. Each single diameter is a decision variable and then it is like a gene in nature. Savic & Walters [15] proved that GA are particularly fit for water distribution network design, since these are complex combinatorial optimisation problems. Moreover, water distribution system designs are usually referred to large scale networks, besides being non-linear and encompassing a number of local-optima. The population-based algorithms are particular efficient for this problems, since they perform a global exploration of the solution domain. In this scenario, the population-based algorithms have a further advantage in comparison with the traditional non-linear constrained programming techniques: they do not require a starting point (set of diameters) close to the best solution that can strongly bias the search for the solution. The main stages of the GA are [12]:

1. Fitness evaluation for the solutions: in water distribution networks the fitness indicators are the economical cost of the network and the constrain on the levels of service (minimum pressure in nodes).
2. The selection of the mating pool: the population is not entirely admitted to the reproduction.
3. Crossover: the genetic information is swapped among the strings representing the diameters of the problem (1).
4. Mutation: randomly chosen genes are mutated, in this way a global exploration of the solutions' space is encouraged.
5. Stop criterion: it is usually based on the statistics on solutions or on the maximum number of generations.

We have to emphasize that the initial population is randomly generated and it is not strongly influencing the final results from a technical viewpoint.

Problem formulation as multi-objective problem

GAs can easily optimise two or more objectives per times, thanks to their particular paradigm based on population of evolving solutions. Therefore, a number of sub-optimal trade-off solutions can be found, in spite of a single optimum for a single objective. On this premise, the construction of a decision support system is advantaged [16]. This multi-objective approach is based on the Pareto dominance criterion as in the remainder [20], see Fig. 1. In water distribution systems, the decision support may be a set of solutions representing the best trade-off between economical costs and levels of service, the latter are considered in a risk-based scenario and then relaxed in comparison with the minimum pressure requirements [3,7]. However, if lower-pressure nodes are accepted, the solution of the design can be significant money-saving. The decision about the best solution, in a decision support scenario, is the task of the decision maker, who estimates the risk and the advantages of each feasible solution in a risk-based scenario of management and planning of water systems [16]. The multi-objective solution for the system (1) is for instance the contemporary optimisation of

$$\begin{cases} f_1\left(\sum\limits_{i=1}^{pipes} C\left(d_{i,j}, L_{i,j}\right)\right) \\ f_2\left(h_i - h^i_{geodetics} - X\right) \end{cases} \tag{2}$$

This example is somehow exemplificative for the description of the potentialities of the multi-objective approach. Further objective functions can be defined, for instance related to the robustness and then to the uncertainty on the demand Q_{nodes}.

In fact, many real-world problems involve simultaneous optimisation of multiple objectives [5,6,15]. Multi-objective optimisation is very different than the single-objective optimisation. In single-objective optimisation one attempts to obtain the best design or decision which is usually the global minimum or the global maximum depending on the optimisation problem is that of minimization or maximization. In the case of multiple objectives there may not exist one solution which is the best global minimum or maximum with respect to all objectives. Therefore, since none of the solutions in the non-dominated set is absolutely better than any other, any one of them is an acceptable solution. The choice of one solution over the other requires the knowledge of the problem and a number of problem-related factors. Thus, one solution chosen by a designer may not be acceptable to another designer or in a changed environment. Therefore, in multi-objective optimisation problems, it may be useful to have knowledge about alternative Pareto optimal solutions [15].

Pareto dominance criterion

There are several approaches to deal with multi-objective optimisation problems. A clear description of these methods is given in Srinivas & Deb [18]. Among the multi-objective strategies, the author's choice is the Pareto dominance criterion [14]. It implies the following main advantages: (1) it is reasonably fast for few objective functions; (2) it deals simultaneously with multiple solutions and (3) it is able to provide a uniformly distributed range of Pareto solutions. The key Pareto concepts are mathematically defined in Van Veldhuizen & Lamont [20], while Fig.1 depicts the concept of Pareto dominance in the case of two objective functions to minimize. The point **A** divides the space R^2 of the functions in four areas. The points in the two areas of "non-dominated" could belong to the same Pareto front of **A** considering minimization both of **F1** and of **F2**. The area of "dominated" points refers to solutions that are worse both considering minimization of **F1** and of **F2**. On the contrary, the area of "dominating" points refers to solutions that are better both considering minimization of **F1** and of **F2**.

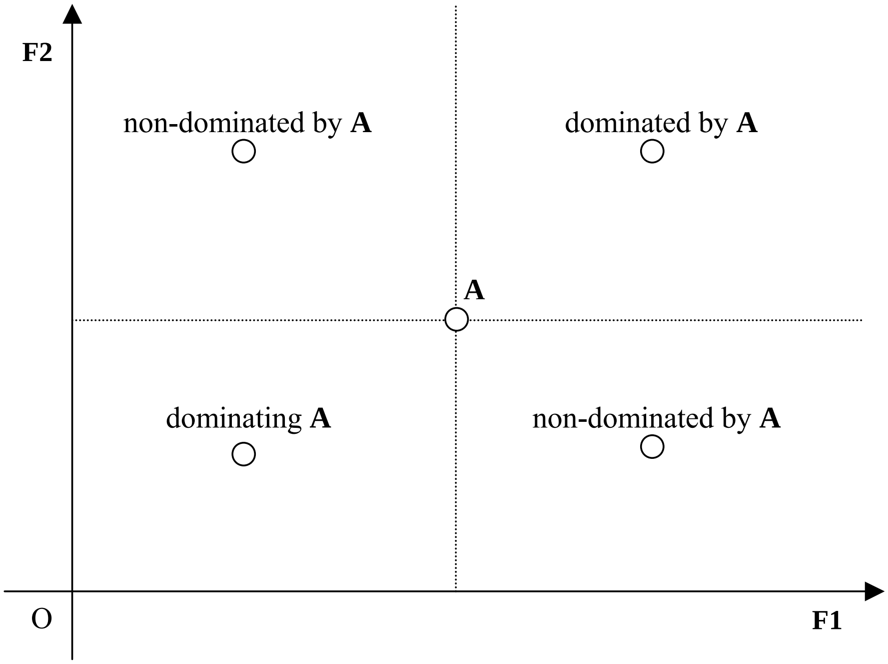

Fig. 1. Pareto dominance criterion considering A.

Uncertainty and Robust formulation for WDS design

When an optimal investment strategy is planned on an assumed period, there are a number of uncertain variables. Some variables of them may be estimated by the know-how of the designer, however the evaluation of variables in a future time horizon of 20-30 years is a hard task. Moreover, there are political, social, etc. components, which increase the complexity related to the uncertainty. In this scenario, the common practice suggests overestimating the uncertain variables; this implies an overestimated or redundant solution, which economically biases the investment plan. Moreover, the redundancy not necessarily decreases the risk on levels [16] of service if the investment is not planned in a scenario of failure probability analysis. For instance, if not strategically evaluated, the redundancy may be used in zones which do not claim for it, thus leaving critical zones uncovered.

On this premise, the innovation in designing water distribution systems is in the development of a strategy capable to return reliable results under uncertainty conditions. This corresponds to introduce the robustness with respect to the future conditions.

The solution of a water distribution network represented by the mathematical system (1) assumes that the water demand is time-invariant as well as the pipe frictions. This is an exemplificative approach, required in order to gain a solution. The water demand is not stable along the network: it changes in the time and space dominion. Also pipe frictions change in time and space along the network. This uncertainty has to be somehow defined, thus featuring the design approach with robustness in the aforementioned management scenario. In Savic

[16] there are some approach addressing this issue. However, there are no considerations about the contemporary optimisation of the economical aspects; Miller *et al.* [13] consider the economical issues related to the robust design. The problem of the robust automatic design was initially faced as single objective problem [10]. In that work, the hydraulic heads in nodes were assumed randomly distributed according to a normal distribution probability function where mean value and standard deviation were given. However hydraulic heads were dependent on further uncertain variables, therefore they need to be considered as dependent variables, i.e. output of functions [17]. There are further reference in literature about the design problem or under uncertainty conditions [1,21]. In particular in Babayan *et al*. [1], a new approach to estimate the uncertainty in water demands in an optimised designing scenario is advanced. In this approach the uncertainty model is integrated in a GA platform which seeks for a robust and economically cheap solution.

The problem of strategic investment planning for the convenient management is therefore tackled by a design approach, which encompasses uncertainty in water demands and pipe frictions. In this scenario, a robust solution to the problem is returned, i.e. a solution able to guarantee stable optimal levels of service when some/all parameters of the system are unsteady and then more robust and effective for management purposes.

The solution

A tool package addressed to the robust automatic design of water distribution networks is demanded to present the following features:

1. A fast and robust base for multi-objective optimisation: OPTIMOGA is used [4].
2. A fast and robust hydraulic simulator for the solution of the water distribution network that is performed a lot of times.
3. A reliable definition of the probability density function related to the uncertain variable (water demand and wall friction of pipes).
4. A Montecarlo sampler of probability density function related to the uncertain variable (water demand and wall friction of pipes).
5. An algorithm for the reduction of the variance in sampling the aforementioned functions.
6. A proper definition and implementation of the objective functions leading the optimisation.
7. Input-output user interface purposed to allow the user to easily evaluate the results and to support the decision.

On this premise, the paper intend to carry on:

1. Modifications of OPTIMOGA, which is now very efficient in comparison with other state-of-the-art genetic algorithms [4]. In particular, a refinement of the mutation operator will be done. Therefore, troubles such as the slow-finish, reported in literature are supposed to be avoided.
2. Implementation of a hydraulic solver based on the algorithm of Todini-Pilati [19], which involves reordering algorithms in order to minimize the band of the node matrix, which is usually sparse. Algorithms such as Column permutation, Column approximate minimum degree permutation, Symmetric reverse Cuthill-McKee permutation, Symmetric approximate minimum degree permutation will be tested.
3. The use of the Beta function:

$$y = \frac{x^{a-1} \cdot (1-x)^{b-1}}{beta(a,b)} \tag{3}$$

   which is defined in the range [0,1] of the x variable. Therefore, the proper settings of y, a and b will return distribution of assumed mean value, variance and shape. For instance, the Beta function where a=b=1 is given, returns an uniform probability density function. If a=b a sort of bounded Gaussian function is returned.

4. A probability density function generator based in rejection and inversion methods [2]. The Latin Hypercube [11], which is a variance reduction methodology. It generates a Latin Hypercube sample X containing n values on each of p variables. For each column, the n values are randomly distributed with one from each interval (0, 1/n), (1/n, 2/n),..., (1-1/n, 1), and they are randomly permuted. Latin Hypercube designs are useful when a sample that is random but that is guaranteed to be relatively distributed according a specific probability distribution function (i.e. Gaussian and Uniform) over each dimension are needed. The Latin Hypercube is used in practical problems, e.g. in uniform distributions it reduce the variance in sampling the Beta function. A massive diminution of the variance implies supported by a proper reduction methodology, significantly decreases the samples' number thus advantaging the efficiency and the run time of the objectives of the problems. This constitutes a great advantage in large scale water distribution networks.
5. The study of objective functions in 2 or more objectives scenarios, which include the economical costs, the minimum pressure levels in nodes related to a pre-assumed confidence, the confidence level as objective itself, the number of critical nodes, etc..

References

1. Babayan, A.V, Kapelan, Z., Savić, D.A. & Walters, G.A., 2004, "Least Cost Design of Robust Water Distribution Networks Under Demand Uncertainty". ASCE Journal of Water Resources Planning and Management, in press.
2. Devroye, L., 1986, "Non-Uniform Random Variate Generation". Springer-Verlag.
3. Giustolisi, O., Laucelli, D., & Doglioni, A., 2004, "Progettazione Automatica e Ottimizzazione Multi-Obiettivo per Reti di Distribuzione". Atti del XXIX Convegno di Idraulica e Costruzioni Idrauliche, Tema C2, N.6, Trento, Italia, 263-270.
4. Giustolisi, O., Doglioni A., Savic, D.A. & Laucelli, D. 2004, "A proposal for an effective multi-objective non-dominated genetic algorithm: the OPTimised Multi-Objective Genetic Algorithm: OPTIMOGA". Report 2004/07. School of Engineering, Computer Science and Mathematics, Centre for Water Systems, University of Exeter.
5. Giustolisi, O. & Doglioni, A., 2005, "Water distribution system failure analysis". International Conference on Computing and Control in the Water Industry, Exeter, UK, September. Accepted.
6. Giustolisi, O. & Laucelli, D., 2005, "A decision support framework for short-time planning of rehabilitation". International Conference on Computing and Control in the Water Industry, Exeter, UK, September. Accepted.
7. Giustolisi, O. & Doglioni, A., 2005, "Progettazione Automatica Robusta di una Rete Acquedottistica in Scenario di Incertezza". I Convegno Nazionale di Idraulica Urbana, Sorrento, Italy, September. Accepted.
8. Goldberg, D.E., 1989, "Genetic Algorithms in Search, Optimization and Machine Learning", Addison Wesley Publishing Company, January.
9. Holland, J., 1975, "Adaptation in natural and artificial systems", The University of Michigan Press, Ann Arbor, Michigan, US.
10. Lansey, K.E., Ning Duan Mays, L.W. & Yeou-Kung Tung, 1989, "Water Distribution System Design under Uncertainties". ASCE Journal of Water Resources Planning and Management, **115**(5), 630–645.
11. McKay, M.D., Conover, W.J., & Beckman, R.J., 1979, "A Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output from a Computer Code". Technometrics, 211, 239-245.
12. Michalewicz, Z., 1994, "Genetic Algorithms + Data Structures = Evolution Programs". AI Series, Springer-Verlag, New York, US.
13. Miller, I., Kapelan, Z. & Savić, D.A., 2001, "GAasset: Fast Optimisation Tool for Strategic Investment Planning in the Water Industry". In Proc. of 4th International Conference on Water

Pipeline Systems, York, UK: 28-30 March, A. Lowdon (ed.), BHR Group, Cranfield, UK, 277-292.

14. Pareto, V., 1896, “Cours D'Economie Politique”. Lausanne.
15. Savić, D.A. & Walters, G.A., 1997, “Genetic Algorithms for the Least-cost Design of Water Distribution Networks”, ASCE Journal of Water Resources Planning and Management, **123**(2), 67-77.
16. Savic, D.A., 2002, “Single-objective vs. Multi-objective Optimisation for Integrated Decision Support”, Rizzoli, A.E. and A.J. Jakeman (Eds.), Integrated Assessment and Decision Support, Proceedings of the First Biennial Meeting of the International Environmental Modelling and Software Society, 1, 7-12.
17. Savić D.A., 2004, “Risk and robust strategic investment planning in the water industry, an Optimisation Approach”, Proceedings of the 4th international conference on decision making in urban and civil engineering, Oporto (Portugal), 28 – 30 October 2004.
18. Srinivas, N. & Deb, K., 1995, “Multiobjective Optimisation Using Nondominated Sorting in Genetic Algorithms”. J. of Evolutionary Computation, **2**(3), 221–248.
19. Todini, E. & Pilati, S., 1989, “A gradient algorithm for the analysis of pipe networks”, Vol.1, System Analysis and Simulation, ed. by Brian Coulbeck and Chun_Hou Orr, Research Studies Press LTD, UK.
20. Van Veldhuizen, D.A. & Lamont, G.B., 2000, “Multiobjective Evolutionary Algorithms Analyzing the State-of-the-Art”, Evolutionary Computation. **8**(2), 125-144.
21. Xu C. & Goulter I.C., 1999, “Reliability-Based Optimal Design of Water Distribution Network”, ASCE Journal of Water Resources Planning and Management **125**(6), 352–362.