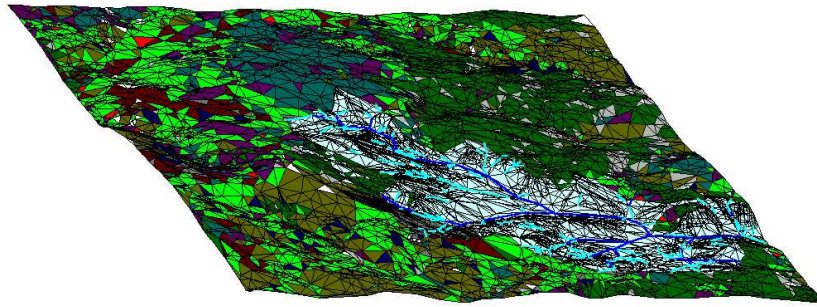




# AN OBJECT-ORIENTATED APPROACH TO HYDROLOGICAL MODELLING BASED ON A TRIANGULAR IRREGULAR NETWORK



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## **Statement of Originality**

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I declare that this dissertation represents my own work, and that where the work of others has been used it has been duly accredited. I further declare that the length of the components of this dissertation is 5500 words for the Research Paper and 9600 words for the Technical Report.

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### Abstract

Traditional grid-based techniques for hydrological modelling have certain disadvantages including (a) that landscapes must be represented at a constant spatial resolution, regardless of wavelength variability in the landscape; (b) that drainage directions are often restricted to 45° intervals; and (c) that geometrical artifacts of the regular grid-structure may be visible in the model output. It is widely thought that variable-resolution irregular data structures would allow more realistic hydrological modelling to be performed upon them than do traditional regular grid-based data structures. A successful and widely used implementation is a triangular irregular structure (TIN).

This project discusses and explores issues of data representation for hydrological modelling. It uses an object-orientated approach to build an adaptive and ‘intelligent’ landscape model, which incorporates a TIN-based data structure and embedded methods and behaviours. A TIN-based data model is used to try and address some of the inadequacies of rasters. Its embedded methods and behaviours allow it to build, maintain, derive its topology and derive hydrological information about itself. It offers an alternative approach to hydrological modelling – one that is defined by its data. TINMOD is such an implementation, designed as part of this study. TINMOD can build itself from raster data input, and delineate its basins. Outputs of TINMOD are presented and discussed. Its performance and sensitivity are evaluated in the context of wider hydrological modelling issues.

## 1 Introduction

The aim of this project is to design, implement and discuss an alternative approach to hydrological modelling, to traditional raster-based models. This alternative approach uses an irregular data structure and object-orientated principles to produce a model which is an adaptive and ‘intelligent’ terrain model, into which both *data* and hydrological modelling *behaviours* are embedded. Its data structure is based on a triangular irregular network (TIN), which may have benefits over regular grid-based data structures for representing elements relevant to hydrological modelling. ‘TINMOD’ is an implementation of this, designed as part of this study. TINMOD has been written in Object Pascal, the code and documentation of which is provided in the Technical Document (Slingsby, 2002).

The motivation for this research is the inadequacies of regular data models for terrain-based environmental modelling, for example as observed by Braun and Sambridge (1997). Tucker *et al.* (2001) list the following as common disadvantages of using rasters: (a) that landscapes must be represented at a constant spatial resolution, regardless of wavelength variability in the landscape; (b) that drainage directions are often restricted to 45° intervals; and perhaps the most serious, (c) that geometrical artifacts of the regular grid-structure may be visible in the model output. In addition, rasters are unable to preserve the arbitrary locations of point data nor linear data without significant data redundancy (Jones *et al.*, 1994). Jones *et al.* (1990) note that advances in computing have made it possible to implement more sophisticated models, capable of representing terrain surfaces more accurately and deal with terrain more

intelligently. In spite of this, rasters are still the most common data model, because of their simplicity (Tucker *et al.*, 2001). Increases in computational speed and storage capacity tend to be used to represent terrain at increasingly higher spatial resolutions, rather than dealing with terrain more intelligently.

The most commonly used structure for terrain modelling after rasters are Triangular Irregular Networks (TINs) (Weibel and Heller, 1991). TINs represent a terrain surface as a mesh of tessellating triangles defined by their vertices as irregularly-spaced point data. The landscape is discretised into irregularly-sized and -shaped spatial units. In addition, linear features can be represented as edges in the same model (Jones *et al.*, 1994; Weibel and Heller, 1991; Dæhlen, *et al.*, 2001) and of particular interest, river networks can be represented as a connected series of edges (Jones *et al.*, 1990). TINs have been used as the basis for landscape-based environmental models by Jones *et al.* (1994); Dæhlen, *et al.* (2001); Tucker *et al.* (2001) and others.

Tucker *et al.* (2001) used object-orientated techniques to implement a TIN model for geomorphological modelling. Using C++, an adaptive and ‘intelligent’ TIN-based landscape was built, with the capability of deriving information about itself and implementing geomorphological and hydrological modelling behaviours. TINMOD uses the same approach as this, but flow is dealt with using a method more suited to hydrological modelling (see section 2.2.2). The object-orientated approach, allows other attributes to be attached to elements of the TIN, for example, TINMOD allows land-cover data to be attached to triangles in the TIN.

## 2 Background

It is appropriate to review the data models commonly used for representing landscapes for hydrological modelling, with their relative strengths and weaknesses for this purpose.

### 2.1 Digital Terrain Models

Digital terrain models (DTMs) are representations of the Earth’s surface. DTMs may contain a variety of terrain information, such as land cover information (Borough, 1986), but the major component is a digital elevation model (DEM) providing the topography. DEMs sample continuous surfaces by discretising them into regular or irregular spatial units and associating each with a value.

Rasters are regular grid-based data models, which discretise the landscape into grids of (usually uniform and square) regular spatial units. This is the most common type of discretisation because it is the simplest data structure (position and topology is implicit) and is easily mapped onto arrays making it computationally efficient (Moore *et al.*, 1991). Also, much of the DEM data now comes in this format directly from air- or space-borne scanners.

Irregular data structures use irregular-sized and/or -shaped spatial units. The motivation for much of the work on irregular data structures has been an attempt to design a more ‘natural’ way of representing data (Moore *et al.*, 1988) – as meaningful

functional units rather than the rather arbitrary units of regular data structures. The two most common irregular discretisation methods are triangular- and contour-based networks (Moore *et al.*, 1991). Triangular irregular networks (TINs) are meshes of tessellating triangular spatial units (facets), defined by their vertices. Vertices usually sample surface-specific points, such as peaks, pits, ridges and channels and the triangle surfaces interpolate the elevation between vertices. Contour-based methods (Moore *et al.*, 1988) use digitised contour lines, represented as strings of coordinates. These, together with streamlines, are used to divide the area into irregular polygons. Band (1989) takes a similar approach by presenting the landscape as a set of functional units, such as slopes and catchments, each with their own hydrological response.

TINs have attracted the most attention because of their simplicity and the local nature of their triangulation (Delaunay) and they are the choice of data model in this project. TINs are created from irregular point data, ideally original survey data. Since most DEMs have already been sampled as rasters, TINs are often resampled from these. Four algorithms for selecting significant points to sample from rasters have been reviewed by Lee (1991): (a) skeleton method; (b) VIP (or filter method); (c) hierarchy method; (d) heuristic method. Variations of these have been implemented and evaluated.

## 2.2 Data Models for Hydrological Modelling

The inadequacies of rasters for hydrological modelling have been observed by many researchers. For example Mark (1978) states that the data model should be chosen on the basis of natural processes of the model, rather than factors such as the data source or the computer hardware.

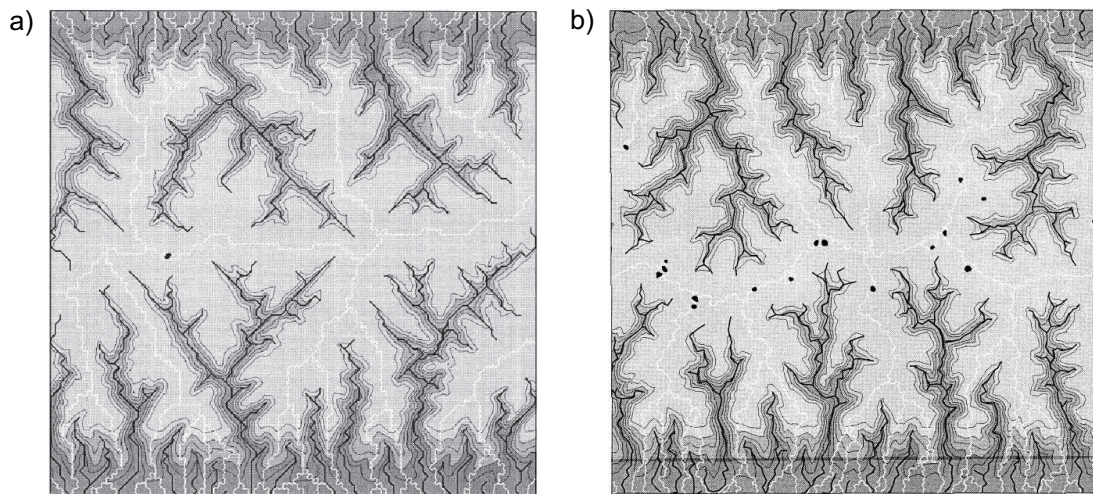
### 2.2.1 Spatial Resolution

Rasters have a uniform spatial resolution regardless of the variability of wavelength of the natural environment. Since the whole landscape must be at the maximum spatial resolution required for any local feature, large amounts of data redundancy occur. TINs allow variable resolution by varying the density of triangle vertices according to local terrain complexity. Long-term landscape models may even have a *temporally dependent* spatial resolution (Braun and Sambridge, 1997).

### 2.2.2 Drainage Directions and Network Representation

Braun and Sambridge (1997) ran a landscape evolution model on both a regular (raster) and an irregular (TIN-based) data structure. The model outputs in figure 1 show that the output of the model which was run on a regular grid has a channel network which is strongly controlled by the regular geometry of the grid, producing unnatural-looking symmetry in the channel network. This difference is due to the way drainage directions are handled.

A good review of the methods for deriving drainage direction on rasters is by Galant and Wilson (2000). The most common method for deriving the drainage direc-



**Figure 1:** Regular and irregular data models compared. (a) Landscape evolution model output, performed on a regular grid. The simulated river channels have a shape strongly controlled by the regular geometry of the grid. (b) Output of the same simulation, but performed on an irregular TIN-based structure. Channel networks have a more ‘organic’ and natural-looking geometry. *Source: Braun and Sambridge (1997).*

tion, ‘D8’ (O’Callaghan and Mark, 1984), only allows flow to enter one of its eight neighbours. This leads to drainage directions being restricted to  $45^\circ$  increments. The ‘Rho8’ (‘randomised single-flow-direction’) method (Fairfield and Leymarie, 1991) is a stochastic version of D8. It avoids long, parallel flow paths, producing more realistic-looking channels, but introduces more cells with no upslope connection. The stochastic element leads to flow lines being different every time the model is run (an undesirable property) (Gallant and Wilson, 2000). Lea (1992) routes water according to a best fit plane reflecting the local aspect. Quinn *et al.* (1991) use a multiple flow routing algorithm, tested using the hydrological model TOPMODEL. They partition flow amongst downslope cells by using the unit contour length for each cell (for single direction routing, this would be the cell width). This algorithm gives more realistic distributions of contributing areas, but tends flow tend to diverge in valleys, where channels are usually well-defined. A user-defined threshold is often set to turn off multiple flow routing when the contributing area becomes too large (Gallant and Wilson, 2000). Costa-Cabral and Burges (1994) and their ‘DEMON Stream-Tube Method’, represents flow in two, rather than one dimension, directed by aspect, by fitting contours onto the raster DEM, and using these to direct flow. These algorithms approximate the flow direction of rasters to overcome the problem that cells in a raster do not have a unique flow direction.

In contrast, TIN facets and edges, do (by definition) have a unique flow direction defined by their steepest lines of descent, whose direction, in general, is not subject to the same constraints as those of rasters. Flowpaths over a TIN surface can be computed using the steepest lines of descent of TIN facets, assuming homogeneous roughness and negligible momentum (Jones *et al.*, 1990). Facets represent areal units, so may serve as a basis to model overland flow. Edges, whose two adjacent edges slope



towards each other, define the edge as channel (concave edge), along which channel flow may be modelled.

Tucker *et al.* (2001) use a different, more robust but simplified approach to flow routing, because the method described above is very sensitive to even small inaccuracies. Because there is more than one way to triangulate the nodes, each alternative may yield a different drainage pattern. Instead of representing two types of flow (across facets and along edges), they follow the method of Braun and Sambridge (1997), in which all flow is routed across Voronoi polygons (tessellating polygons centred around TIN nodes whose edges perpendicularly bisect TIN edges). Since channels are not explicitly represented, this approach is more suited to long term landscape evolution rather than to hydrological modelling. This approach could also be used where the input data resolution was in the order of 1km spacing, in which case, there is no point in representing channels explicitly (Chase, 1992; Braun and Sambridge, 1997).

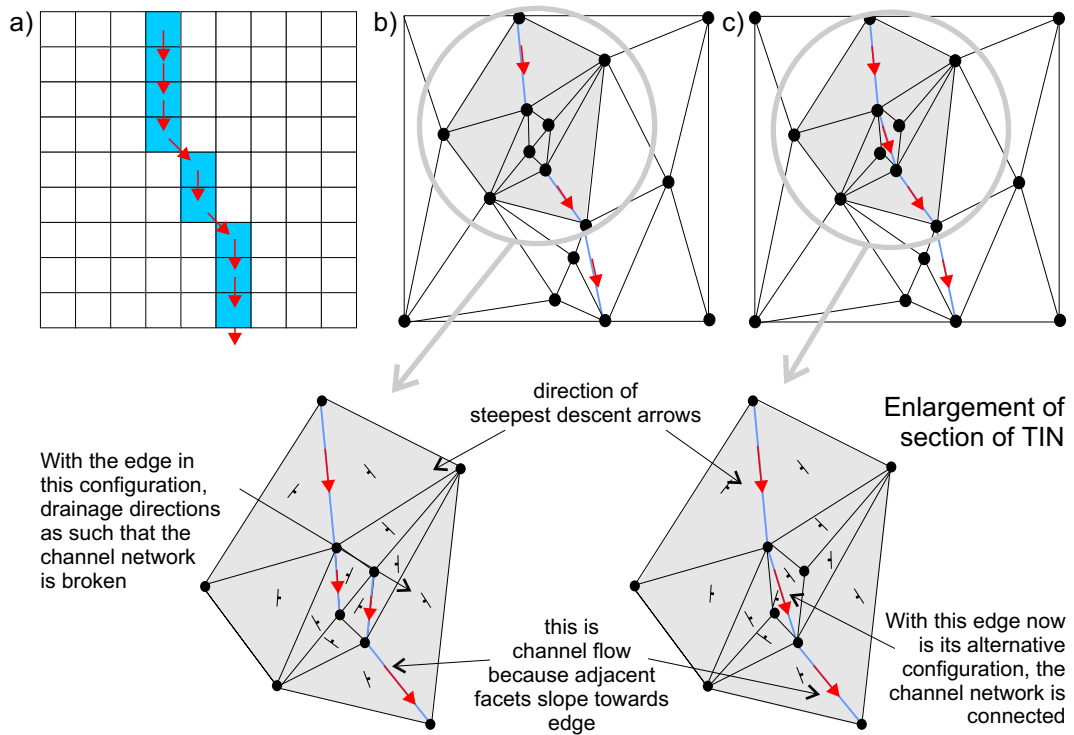
An illustration of the sensitivity of TIN drainage directions to the method of triangulation (using facet flow and edge flow) is illustrated in figure 2, and is discussed in the next section.

### 2.2.3 Representation of Space, Structures and Networks

Since rasters are usually mapped onto arrays, there is usually only one data value associated with each cell. In this case, a decision has to be made as to whether the value represents point data (at grid intersections or cell centroids) or areal data (the cells' extents). A more complex data structure, such as TINs, can allow the representation of areas (facets), lines and networks (edges) and points (vertices or nodes) in the same model, of any size, shape or angle (Jones *et al.*, 1994; Weibel and Heller, 1991; Dæhlen, *et al.*, 2001), including river networks (Jones *et al.*, 1990). Figures 2a and 2c shows how a river might be represented in a raster and a TIN.

When TINs are built, the vertices need to be triangulated. One of the best triangulation schemes is *Delaunay triangulation*, because it tends to produce triangles whose angles are equiangular ('fat' triangles). This makes them local in character and makes interpolation of the elevation of the surface at any point more accurate (Jones *et al.*, 1994). The criterion is that the triangle's circumcircle (a circle passing through all three points), should not encompass any other points. Delaunay triangulation is unique (except in certain special cases, where the vertices of two adjacent triangles form a square (Sloan, 1987)). The uniqueness and local character of Delaunay triangulation is exploited in the 'Implicit TIN' data model Jones *et al.* (1994), in which the TIN is represented as its vertices (point data) only, and sections of the TIN are triangulated only when required.

The problem with always using pure Delaunay triangulation, is that edges cannot be guaranteed to coincide with linear features. Figure 2b shows pure Delaunay Triangulation, where one of the edges does *not* coincide with a river segment, thus a river cannot be represented along the line it ought to be. The enlarged section of figure 2b shows that the local drainage direction may be affected by this, such that flow may not be able to get through. To resolve this problem, certain edges need to be *constrained* (Chew, 1987), thus relaxing the Delaunay criterion for these (Weibel and Heller, 1991).



**Figure 2:** Rivers and drainage directions on rasters and TINs. (a) an example of a river represented on a raster. (b) an example of the same river represented on a TIN. Strict Delaunay triangulation has caused a break in the network. The enlarged section shows the reason for this. The steepest descent directions of the facets are such that channel flow (flow along edges) cannot proceed pass this point. Flow as overland flow (across facet surfaces) can proceed though. (c) The edge has been constrained (forced into this configuration) to coincide with the river, relaxing the Delaunay criterion for this edge). The drainage directions are now such that a fully connected channel network exists.

In figure 2c, this edge has been constrained to coincide with the river channel, which gives the river representation and drainage characteristics required. In many cases, water may bridge a gap in a river network by a small amount of overland flow but this is not ideal.

The hydrological significance of whether flow is represented as channel flow or overland flow, is that are often modelled separately. Channelised flow has a higher velocity due to factors such as a higher hydraulic radius and lower friction; whereas overland flow through vegetation is significantly slowed. Also, for models with constant precipitation input, overland flow will have a significant precipitation contribution (due to its high area:depth ratio); but this input will be insignificant for channel flow. These are the reasons why a fully-connected network is desirable.

Of the four TIN generation algorithms listed in Section 2.1, only the ‘skeleton method’ attempts to identify and extract rivers and channels from terrains to ensure that the triangulation represents these properly. The other algorithms produce unconstrained TINs, where edges may not coincide with linear features. TINMOD uses the ‘skeleton method’ to constrain the Delaunay triangulation.

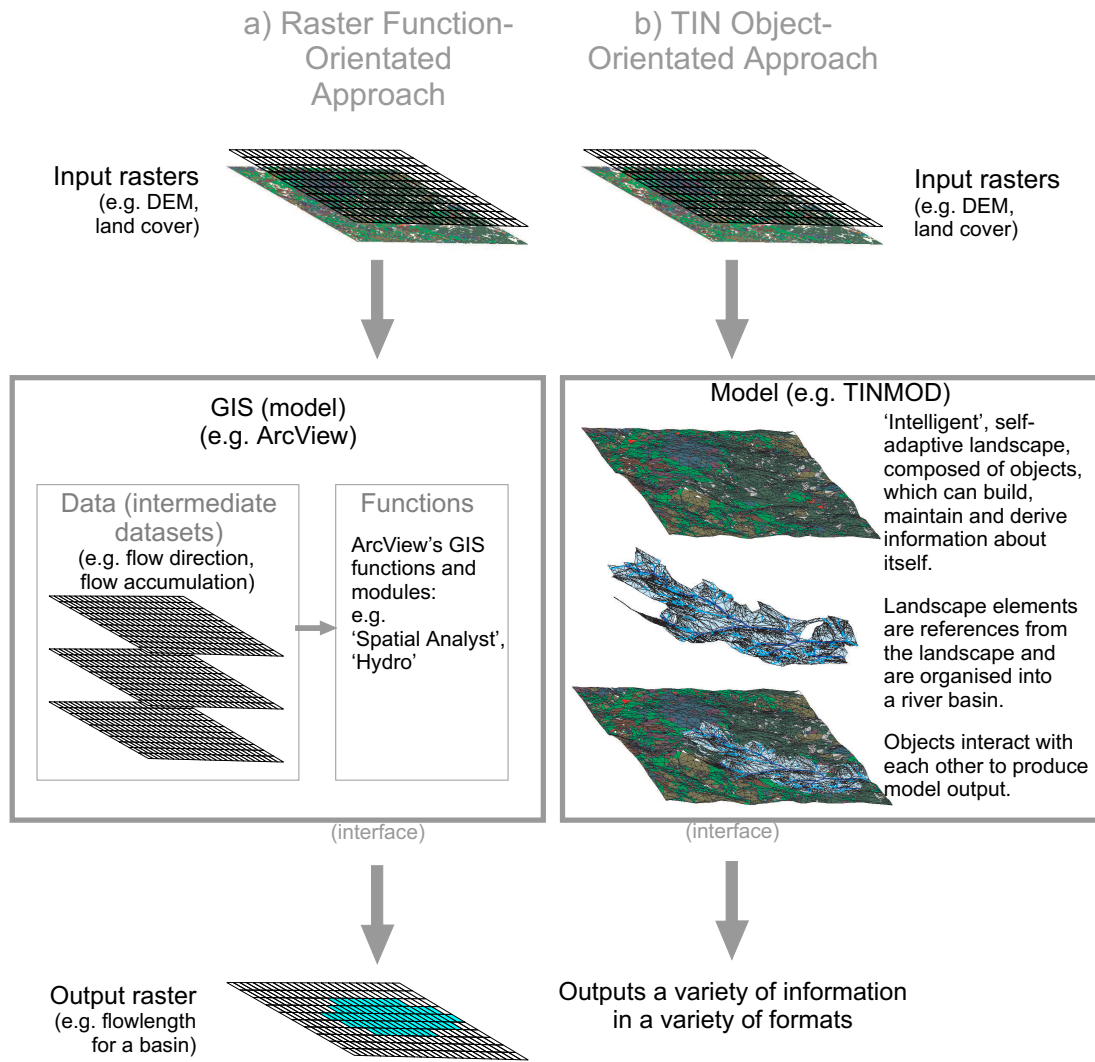
### 2.3 Object-Orientation and ‘Intelligent Landscapes’

The traditional ‘function-oriented’ approach of modelling models a real-world system by mapping it onto rigid data structures, and providing functions which operate on these. The ‘object-orientated’ approach models a real-world system as a series of interacting entities (objects). The real world system is abstracted into objects which contain both *data* and *behaviours* and can interact with each other. Behaviours allow the object to create, maintain and derive information about itself. Figure 3 shows the conceptual difference between the traditional approach (where the data and the algorithms are separated) and the object-orientated approach (where the data its relevant routines and algorithms are held together as an ‘object’). The object-orientated approach to design modularises the program, allowing more complex data models and concepts to be implemented with greater ease. Its embedded behaviours make it (a) adaptive (it can cope with dynamic change), (b) ‘intelligent’ (it can provide complex topological information about itself) and (c) ‘hydrologically aware’ (it can model hydrology upon itself).

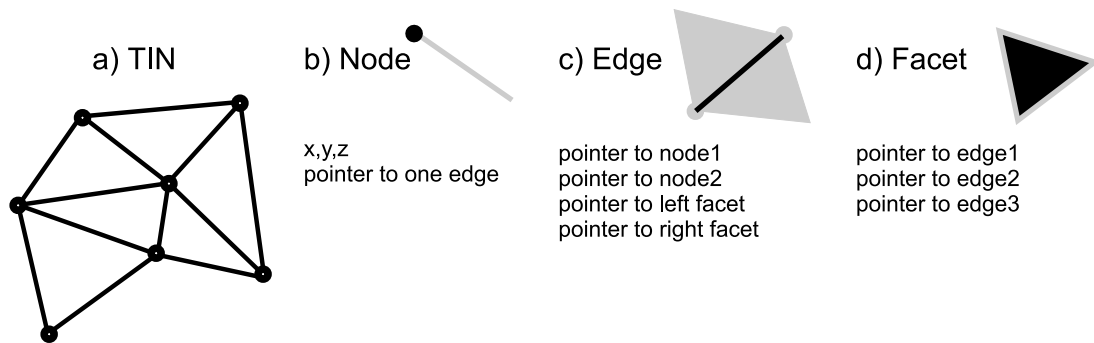
TINMOD is an ‘intelligent landscape’ model, which abstracts landscape into TIN element objects (nodes, edges and facts). Each is ‘responsible’ for building, maintaining and deriving information about itself. The model is the interaction of these objects with each other.

## 3 TINMOD

TINMOD was written in Object Pascal to explore the concepts discussed in this and the previous section. Object Pascal is a high-level, compiled, strongly typed language, that supports object-orientated design (Borland, 1998) and has a particularly good interface with the graphical routines of Microsoft Windows.



**Figure 3:** Raster function- and TIN object-orientated approaches. (a) A grid-based function-orientated approach to modelling. The data model is grid-based, and all the hydrological functions are modelled by the software. (b) The 'intelligent landscape'-based object-orientated approach used by TINMOD. The data is initially imported as rasters, but is incorporated into a data structure which has the ability to build, maintain and derive information about itself. A basin is delineated by storing a branched list of references to parts of the landscape as an object, which has the ability to derive hydrological information about itself.



**Figure 4:** A TIN (a) is made up of nodes (b), edges (c) and facets (d). Topological relations are stored explicitly.

Taking a raster DEM as its input, TINMOD will delineate and draw all the basins and calculate two simple geometrical attributes, area and maximum flowlength. This section describes the concepts used. Further details are discussed in (Slingsby, 2002).

### 3.1 The Data Model

Tucker *et al.* (2001) abstract a TIN into three different object classes, (a) nodes, (b) edges and (c) facets (figure 4). This approach is used as the basis for this project, because a TIN can easily be modularised in this way and it fits in with principles already discussed – for example that edge-flow and facet-flow are dealt with differently. Each of these objects has routines for building itself, deriving its topology and providing hydrological information about itself. These objects interact and work together to produce an overall landscape model with the ability to create, modify and derive information about itself, and derive simple hydrology, using a *piecemeal* approach.

### 3.2 Adaptivity and Derived Topology

The behaviours of the TIN enable it to dynamically adapt to change. For example, if a new node is added or removed, the surrounding TIN elements automatically update their triangulation and stored topology. Most of the topology is derived, so these will be unaffected.

### 3.3 Building a landscape

TINMOD builds landscapes by adding nodes (TIN vertices) to a TIN, one-by-one. As each node is added, the triangulation is updated to ensure it conforms to the Delaunay Triangulation criterion. Four algorithms for selecting points from rasters for this purpose are implemented: (a) skeleton method; (b) VIP (filter method); (c) hierarchy method; (d) heuristic method. These are described in Slingsby (2002, section 6), fully reviewed in Lee (1991) and compared in section 4.1.

The latter three algorithms aim to produce a good overall landscape representation, using only the *shape* of the landscape, within a particular tolerance. As discussed in section 2.2.3 and illustrated in figure 2, while this approach to TIN building can result in good *overall* terrain model, it does not ensure, or even test whether particular *important parts* of the landscape for hydrological modelling are represented properly, specifically whether river channels are fully connected. The ‘skeleton method’ attempts to resolve this using a two stage process (Fowler and Little, 1979). The *first stage* takes a *functional* approach, by identifying the channels and ridges from the original raster. A line-following algorithm follows channels and ridges from pits and peaks, respectively, and then a line-thinning algorithm (Slingsby, 2002, section 6.5) is used to generalise these within a vertical and horizontal tolerance. The resulting points are added to the TIN. It is ensured that each edge coincides with the specific linear feature, and the edge is constrained (made into a breakedge) to *prescribe* its configuration. Once the ridge and channel networks are explicitly represented by fully-connected constrained edges, the *second stage* is initiated, which builds the rest of the TIN around these linear features using one of the other three algorithms to represent the other parts of the landscape within a particular tolerance.

### 3.4 River channels and Representations of Flowpaths

Jones *et al.* (1990) defined ‘channels’ as edges whose two adjacent facets slope *towards* them, and ‘ridges’ as edges whose two adjacent facets slope *away* from them. Pits are nodes with no outlets, whose surrounding edges all slope towards them.

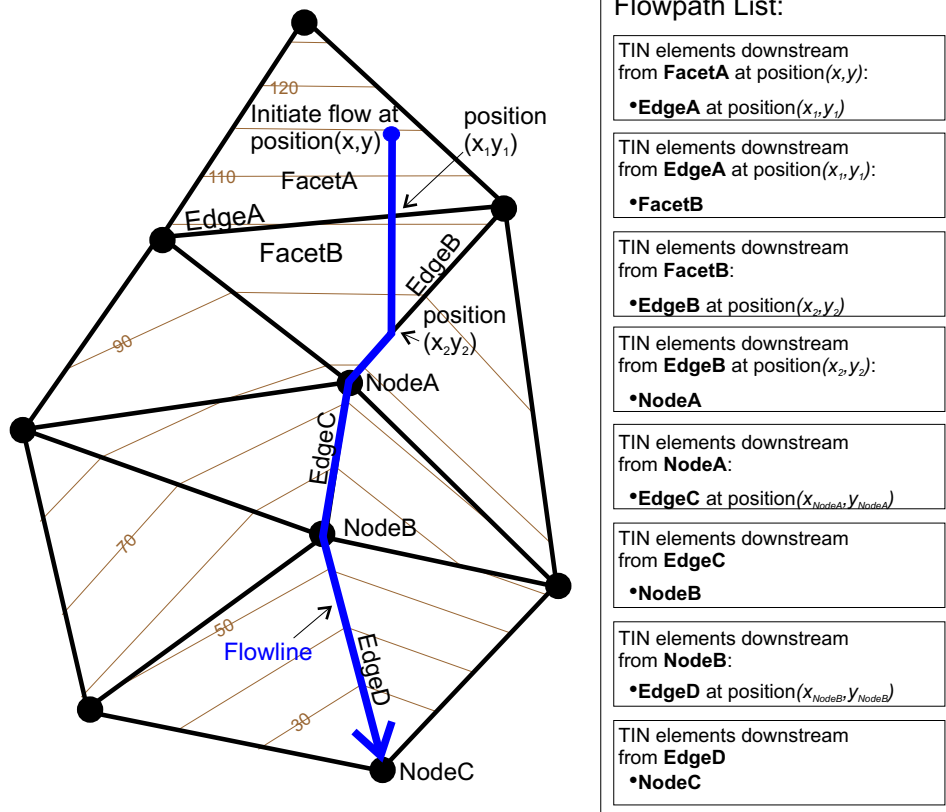
### 3.5 Defining Flowpaths

When defining flowpaths, flow is treated as *lines*. Each TIN element can calculate where flow will leave it, using the ‘rules’ of Jones *et al.* (1990). Flow leaves *nodes* via all the edges defined as ‘channels’ which slope away from it, or via a facet, if its gradient of steepest descent is greater than that of the edges on either side. Flow leaves *edges* via its lowest node if it is defined as a ‘channel’, otherwise flow leaves via the adjacent facet which slopes away from it. Flow leaves *facets* down their path of steepest descent via either the edge or the node that this line intersects.

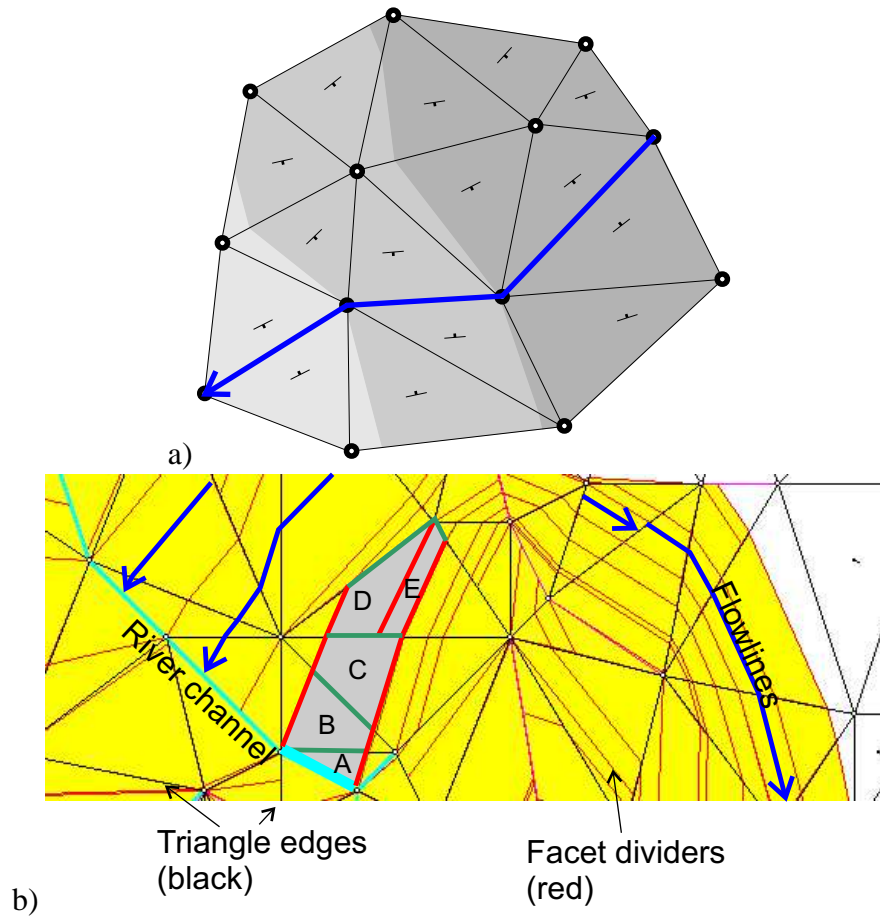
Downslope flowpaths are defined using these rules in a piecemeal fashion from an arbitrary  $x, y$  position, downslope towards a basin outlet (or pit). See figure 5 for a description of the procedure.

### 3.6 Delineating Basins

While ‘flowpaths’ treat all flow as *lines*, for basin delineation flow is treated in two ways. Channel flow (flow along edges) is treated as lines, but overland flow (flow across facets) is treated as *areas*. Channels join up to form river networks and areas of overland flow contribute to downstream channels. Since facets may have more than one output and input, ‘facet dividers’ parallel to the facet’s direction of steepest descent are used to divide the facets into ‘facet areas’, each of which has a unique



**Figure 5:** Deriving flowpaths. Flow is initiated at position  $(x, y)$  on FacetA. FacetA is ‘asked’ via which elements the flow will leave. Using the path of its steepest descent, the next TIN element is EdgeA, intersected at position  $(x_1, y_1)$ . This is returned by FacetA. EdgeA is then ‘asked’ via which elements the flow will leave. EdgeA is not a channel (its adjacent edges do not flow towards it), so flow leaves it via the facet which flows away from it – FacetB. FacetB is ‘asked’ via which which element the flow will leave. By following its steepest descent path from the point at which its previous edge was intersect (position  $(x_1, y_1)$  on EdgeA), EdgeB is found to be the next element and is intersected at  $(x_2, y_2)$ . EdgeB is a channel (its adjacent facets both slope towards it), so flow leaves via its lowest node, NodeA. NodeA has one channel which flows out of it, EdgeC in this case. EdgeC returns NodeB, NodeB returns EdgeD and EdgeD returns NodeC. If NodeC has no outlets, the end of the flowpath has been found. *Diagram based on Jones et al. (1990, p1238).*



**Figure 6:** (a) Basin segments. For each river segment (blue), all the ‘facet areas’ which flow into them are shaded a different shade of grey. ‘Facet areas’ are bounded by steepest-descent lines. (b) Output from TINMOD illustrating this. TIN edges are coloured black. Facets are divided into ‘facet areas’ by up by ‘facet dividers’ (coloured red), such that all a facet area’s input flow leaves by a unique output. The grey area shows a worked example, starting at the river channel. All of the input flow of facet areas ‘A’, ‘B’ and ‘C’, leaves by a unique output (where inputs and outputs are defined as edge segments – shown in green). The area comprising ‘D’ and ‘E’ is divided into ‘D’ and ‘E’ by the facet’s steepest descent path such that each area has a unique output for its input. Although both feed into facet area ‘C’ (via an edge segment), they have a common output from area ‘C’.



output for its inputs. Thus the areas of facet which flow into the each channel segment in question, is explicitly delineated. This is the basis of TINMOD's basin delineation, which actually defines new spatial units for any basin calculation performed, smaller than the facets themselves. This is an important difference between TINMOD and a raster-based model – although TINMOD's spatial units are larger than the spatial units of a raster, they can be operated on at a sub-spatial unit scale. This important concept is illustrated in figure 6 and was briefly mentioned by Jones *et al.* (1990), but is expanded here.

Basins are delineated in a piecemeal fashion, in a similar way to flowpaths, but from a basin outlet of pit (or 'seed' (Jenson, 1991)), each element returning its *up-stream* elements. TINMOD enumerates all the pits in the tin, and can derive a basin from each one. The TIN can have many spurious pit in it, some of which may have come from the original raster (Quinn *et al.*, 1991). Spurious pits need to be removed according to some criteria, because each will have a spurious basin associated with it. TINMOD *dissolves* unwanted basins, by raising the node's height until it ceases to be a pit. The adaptive nature of the TIN dynamically changes its drainage character, and basin area formerly belonging to the raised node will drain into another basin. Many of the spurious basins will be very small, so using a size threshold is a good automatic filter, dissolving all basins below a certain area threshold (but ignoring basins at the edge of the TIN, which may be legitimate partial basins). The main problem with this automatic approach is that it is sensitive to the order in which pits are filled, because the adaptive landscape's local character is changed when a basin is dissolved, affecting the areas of remaining basins. TINMOD arbitrarily fills them from left to right. As subsequent sections show, the automatic size threshold filter *is* very useful, but in some cases, should be used along with, careful manual dissolution of basins (using a contour map as a guide) to yield the most satisfactory results.

### 3.6.1 Calculating Basin Areas

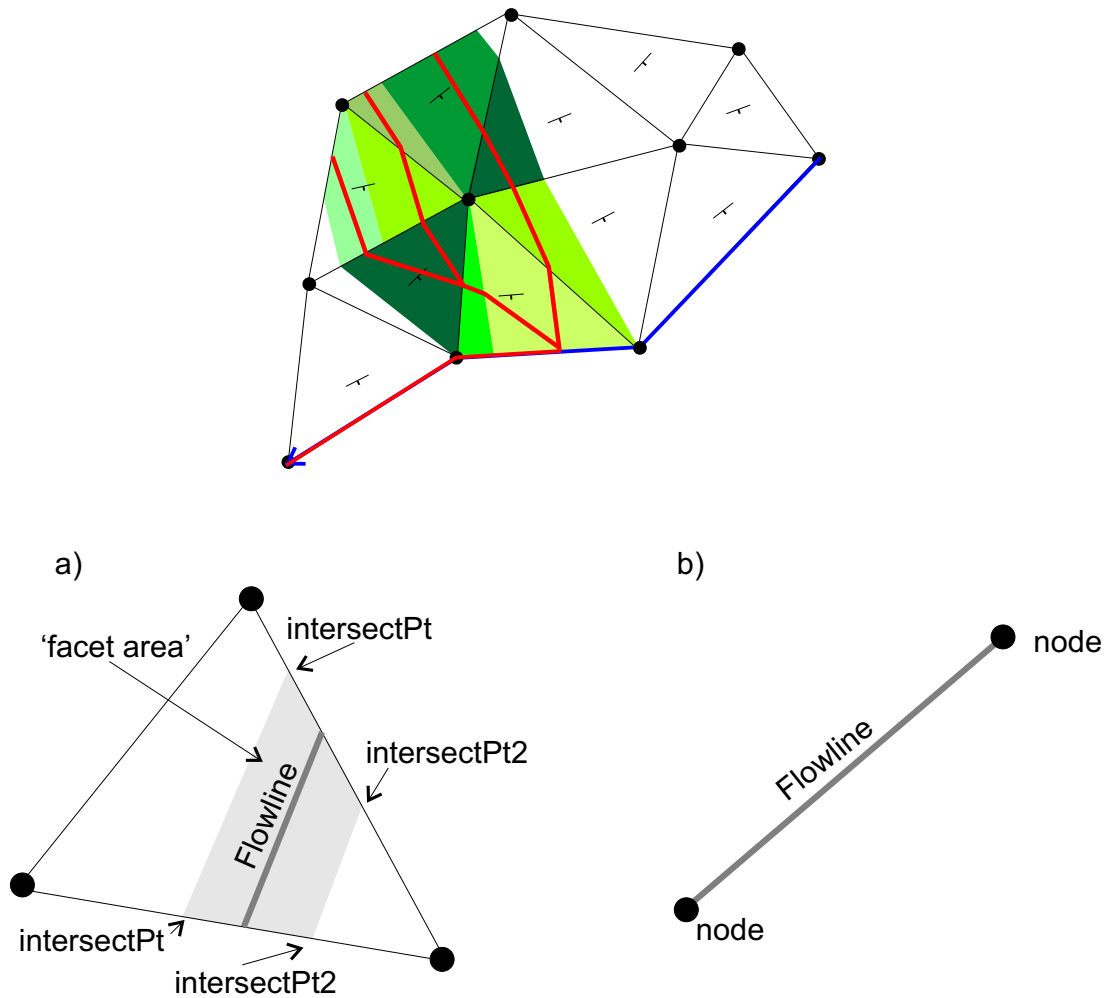
The area of the basin can be calculated by simply summing the areas of all the 'facet areas' from the basin outlet. Since each 'facet area' has a unique output, areas are not counted twice. Where a node has more than one outlet, the upstream area is divided equally amongst these outlets.

### 3.6.2 Calculating the Flowlength

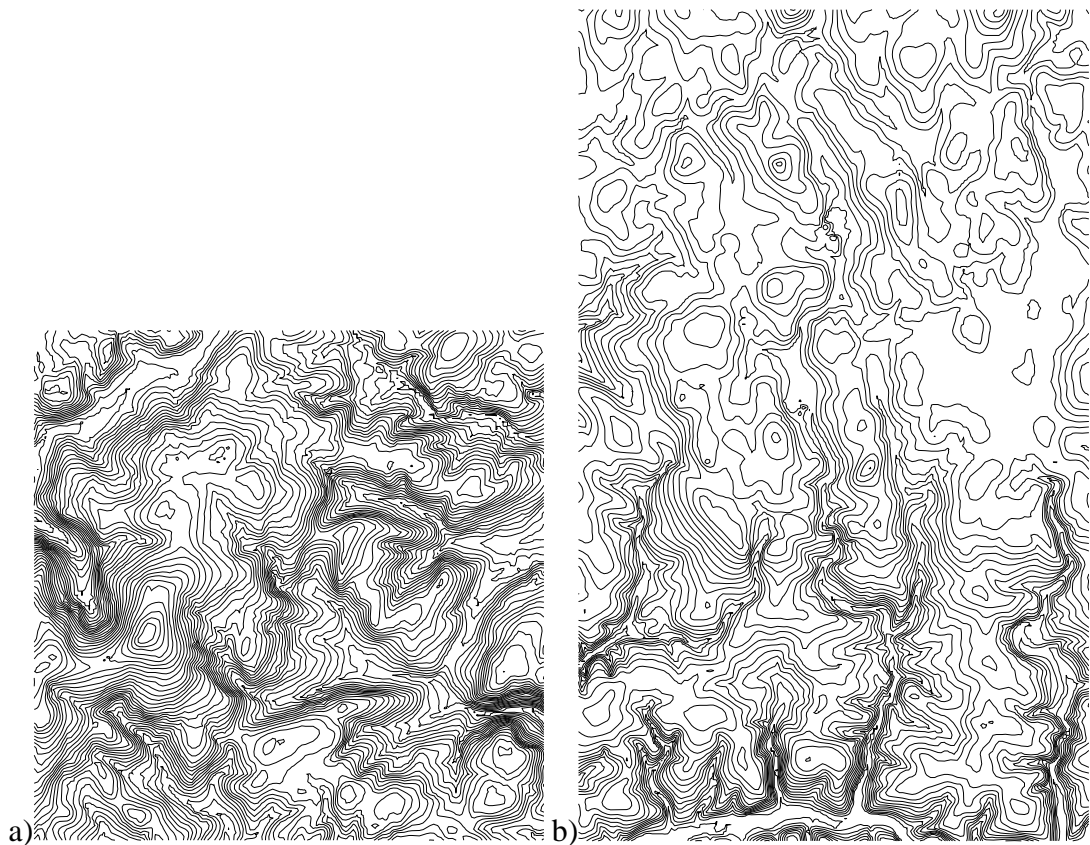
The maximum flowlength is calculated by calculating the lengths all the channels and the lengths across facet areas from the basin outlet to each watershed edge, and taking the maximum. See figure 7.

## 4 Results and Discussion

The models were tested with two DEMs, supplied by the Centre for Ecology and Hydrology. DEM1 is from Mid Wales, in the headwaters of the River Severn and the



**Figure 7:** Calculating flowlengths. (a) Flowlengths the sum of the lengths of each channel and the distance across ‘facet areas’ until the watershed is reached. In this figure, from the lowest node, flowpaths (shown in red) are shown for facet areas shaded in green. (b) Flowlengths across each facet area and along each edge is illustrated.



**Figure 8:** *a) Contour map of DEM1, 7×7km in size. b) Contour map of DEM2, 8×13km in size. Contours generated from raster DEMs by ArcINFO. Based on Spatial Data licensed from the Centre for Ecology and Hydrology, ©CEH. Data is ©Crown Copyright (0186A).*

**Table 1:** Characteristics of TIN point selection algorithms at different tolerances.

DEM1			DEM2		
Algorithm (tolerance)	Points used	Mean error (cm)	Algorithm (tolerance)	Points used	Mean error (cm)
a) Hierarchy(40)	21%	11.47	b) Hierarchy(40)	11%	11.40
Hierarchy(20)	40%	5.07	Hierarchy(20)	22%	5.48
Hierarchy(10)	60%	1.90	Hierarchy(10)	37%	2.49
VIP(40)	11%	101.85	VIP(40)	5%	151.34
VIP(20)	32%	20.79	VIP(20)	14%	65.43
VIP(10)	60%	4.67	VIP(10)	30%	12.09

River Wye. DEM2 is in Trengoffe, Cornwall (see figure 8). Measurements and observations are not available to *verify*, but an attempt will be made to *validate* it (Oreskes *et al.*, 1994) by comparing with a simple model implemented in ArcView and by visually comparing watersheds and drainage directions with the contour maps. TINMOD's characteristics will then be explored.

## 4.1 Evaluation of TIN point selection algorithms

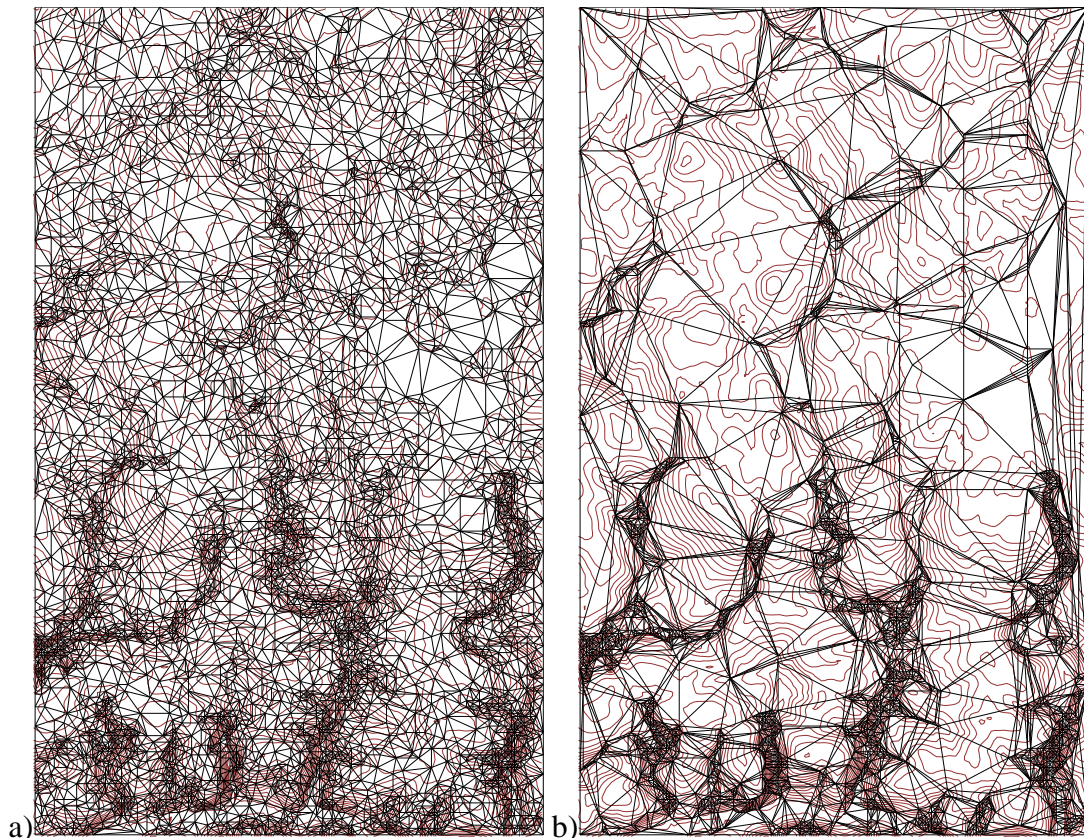
The VIP, hierarchy method and heuristic methods (section 3.3) are evaluated here for their suitability as the second stage of the skeleton method for producing constrained TINs, and alone for producing unconstrained TINs.

It was found that the 'heuristic method' as implemented by TINMOD is unsuitable due to the large amount of time it takes (Slingsby, 2002, section 6.4). The remaining two methods are compared visually and using 'mean error', an overall measure of how well it fits to the surface. Mean error is calculated as follows. For each point in the original raster, its deviation as an *absolute* value (treating them all as positive) from the interpolated surface of the TIN at the same  $x, y$  point is calculated. These are summed and then divided by the number of points in the TIN.

The tolerances for the different algorithms are not directly comparable (see (Slingsby, 2002, section 6) for details). Table 1 shows the numerical characteristics of the algorithms (the tolerance is given in brackets) and figure 9 shows how the Hierarchy(40) method compares with the VIP(10) method for DEM2 (both have similar mean errors).

The percentage of points uses by the algorithms depends on the form of the landscape, but table 1 shows that the VIP method uses more points to produce a landscape than the hierarchy method. Figure 9 shows that the local character of the VIP method overrepresents breaks of slope and steep slopes, and underrepresents all other part of the TIN, producing a stark dichotomy of spatial scales. The hierarchy method produces a more balanced-looking TIN.

These tests suggest that for most purposes, the hierarchy method with a tolerance of 40cm is the most suitable for this application. Unless otherwise stated, the 'hierarchy40' method is used with the 'skeleton method' to produce constrained TINs and alone to produce unconstrained TIN.



**Figure 9:** Comparison of DEM2 triangulated by using two point selection algorithms. (a) The hierarchy method with a tolerance of 40cm. (b) The VIP method, with a tolerance of 10cm. Although these have difference tolerances, table 1 shows that they have a mean error. Note that the VIP method *overrepresents* areas of complex topography (i.e. the river channel) and *underrepresents* simpler topography (hillslopes). *Contours generated from raster DEMs by ArcINFO. TINs generated by TINMOD. Based on Spatial Data licensed from the Centre for Ecology and Hydrology, ©CEH. Data is ©Crown Copyright (0186A).*

## 4.2 Model Validation

Constrained TINs were used for these tests. In TINMOD's outputs maps show, constrained channels as *dark blue* lines, and *light blue* TIN edges for unconstrained channels (where channels were not extracted).

### 4.2.1 Comparison with an ArcView Model

These tests were run to assess whether TINMOD produced comparable results to a widely used raster-based approach to modelling, though the comparison is made in the light of the issues discussed about some of the inadequacies of raster-based representation for hydrological modelling.

Figures 10 and 11 show watersheds delineated in both ArcView and TINMOD. ArcView delineated basins greater than 5km<sup>2</sup> and TINMOD automatically dissolved basins of less than this area (3.7km<sup>2</sup> for DEM1). Tables 2 and 3 show their characteristics. For DEM1, these are very similar. For DEM2, only basins 1, 3 and 6 are directly comparable, and these characteristics are again very similar. TINMOD's basin 8 is several of ArcView's basins merged.

From these results, few conclusions can be drawn, but they give legitimacy to TINMOD basin calculation algorithms and it is encouraging that two different methods of basin delineation can yield similar results.

### 4.2.2 Comparison with Contour Maps

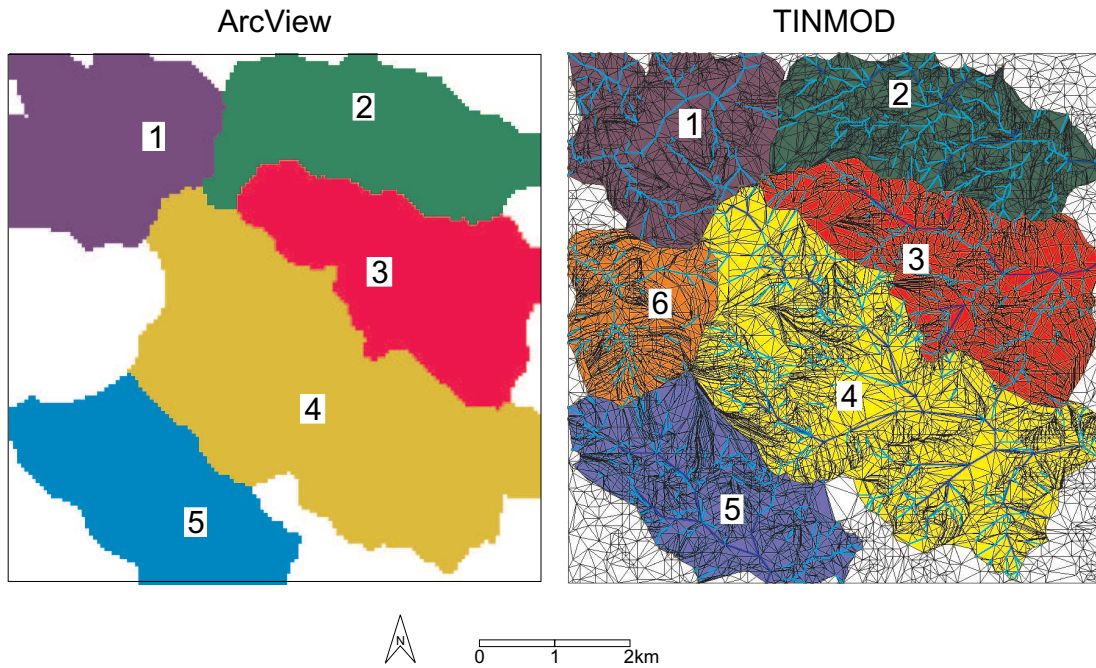
TINMOD's basins were overlain with the contour maps of figure 8, the results of which can be seen in figures 12 and 13. They both show a very good fit to the contour maps (see parts *c* of the figures).

In DEM2 (figure 13), basin 8's outlet is not at the edge of the TIN. Inspection of the contour map shows that its outlet is delineated from a tributary of an extremely large channel in the south, which has been ignored. Since this channel's basin is only partially visible, it is better to use its sub-basin. However the reason it was ignored is that algorithm used in the 'skeleton method' for channel extraction did not use the channel, because it found no pit to follow it from. It is a localised, raster-based algorithm which defines a cell as a pit if it is the lowest of its eight neighbours. If a pit cannot be recognised in the centre of a 150m×150m area (for 50m grid), then TINMOD will fail to extract it. This is a question of scale; the pit may have been too large, for example in the bottom of a large, flat-bottomed, U-shaped glacial valley. Semi-automatic methods can be used to extract this channel if required, see section 4.4.

These results show that for these two DEMs, TINMOD produces reasonable basins, which coincide very well with structural features of the landscape.

## 4.3 Constrained and Unconstrained TINs

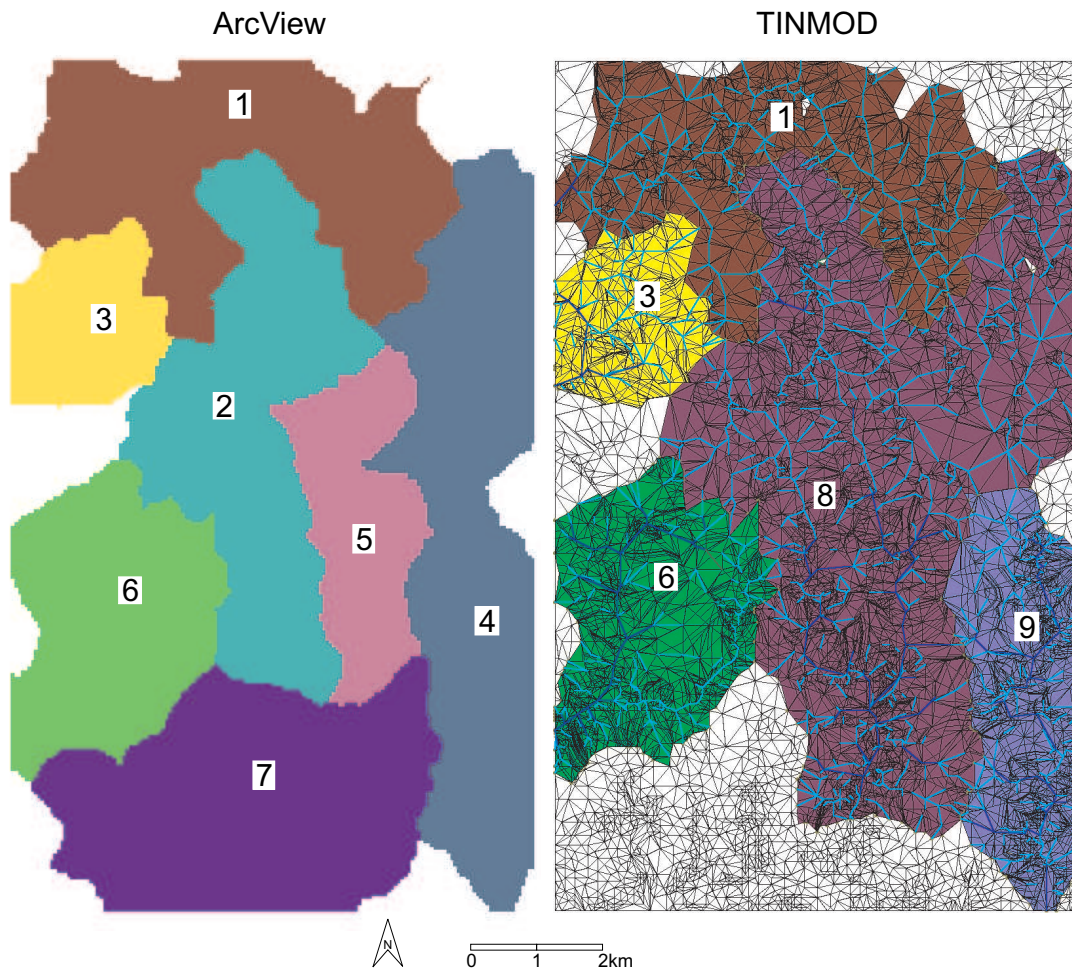
As discussed in section 2.2.3, constrained TINs should be better at representing fully connected river networks on TINs. This section tests whether this makes a practical difference to the model outputs.



**Figure 10:** Watersheds for DEM1 delineated by ArcView (basins of more than 2000 cells or  $>5\text{km}^2$ ) and TINMOD (basins  $>3.7\text{km}^2$ ). The extra basin delineated by TINMOD is due to the smaller threshold).

**Table 2:** Attributes of DEM1’s basins (figure 10).

ArcView			TINMOD		
Basin	Area (km <sup>2</sup> )	Flowlength (km)	Basin	Area (km <sup>2</sup> )	Flowlength (km)
1	6.44	5.98	1	5.61	4.27
2	5.88	4.81	2	6.40	6.21
3	6.92	4.45	3	6.49	5.75
4	13.66	7.84	4	13.85	7.67
5	5.97	4.02	5	4.98	4.02
—	—	—	6	3.74	3.33

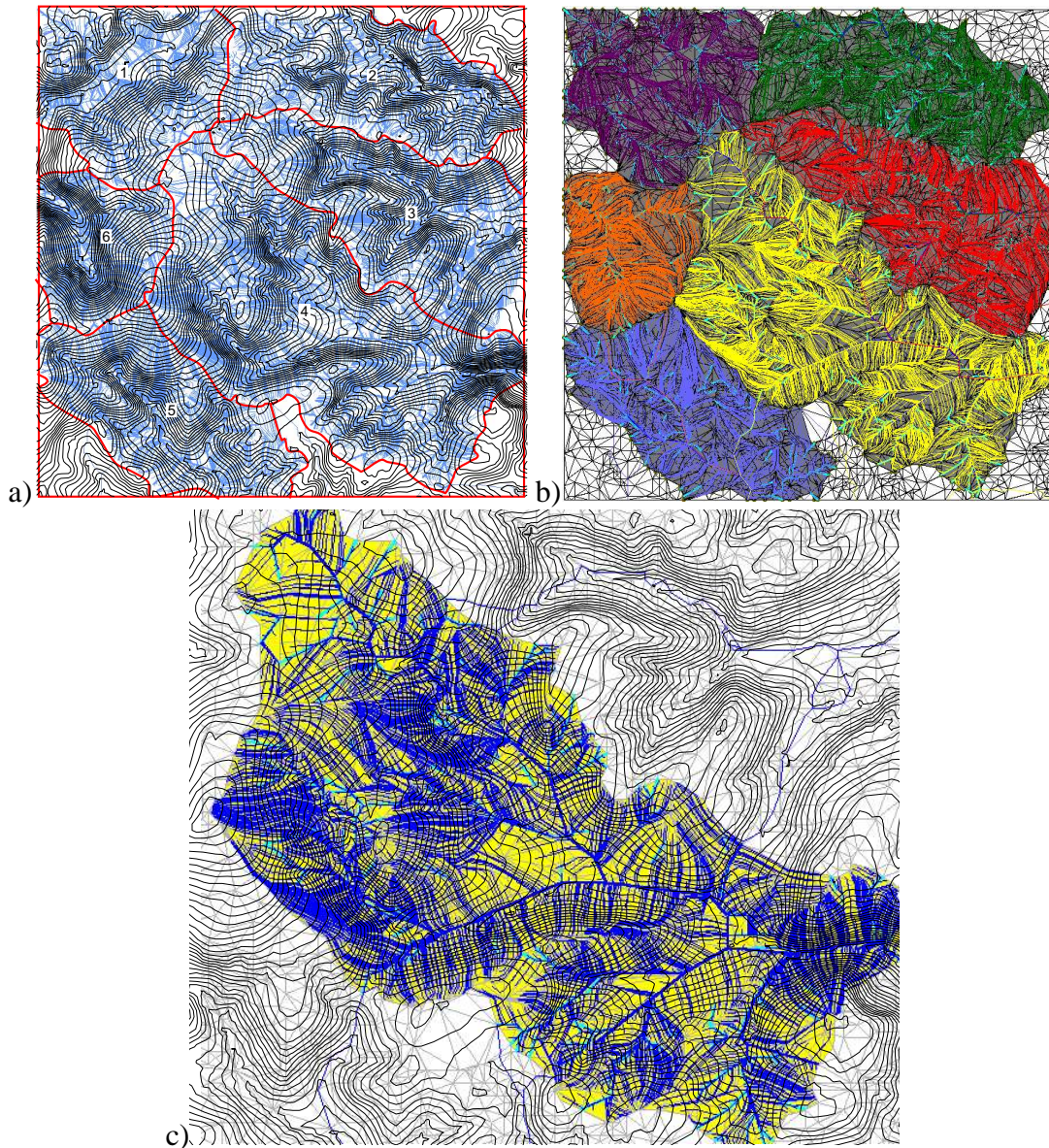


**Figure 11:** Watersheds for DEM2 delineated for ArcView (basins of more than 2000 cells or  $>5\text{km}^2$ ) and TINMOD (basins  $>5\text{km}^2$ )

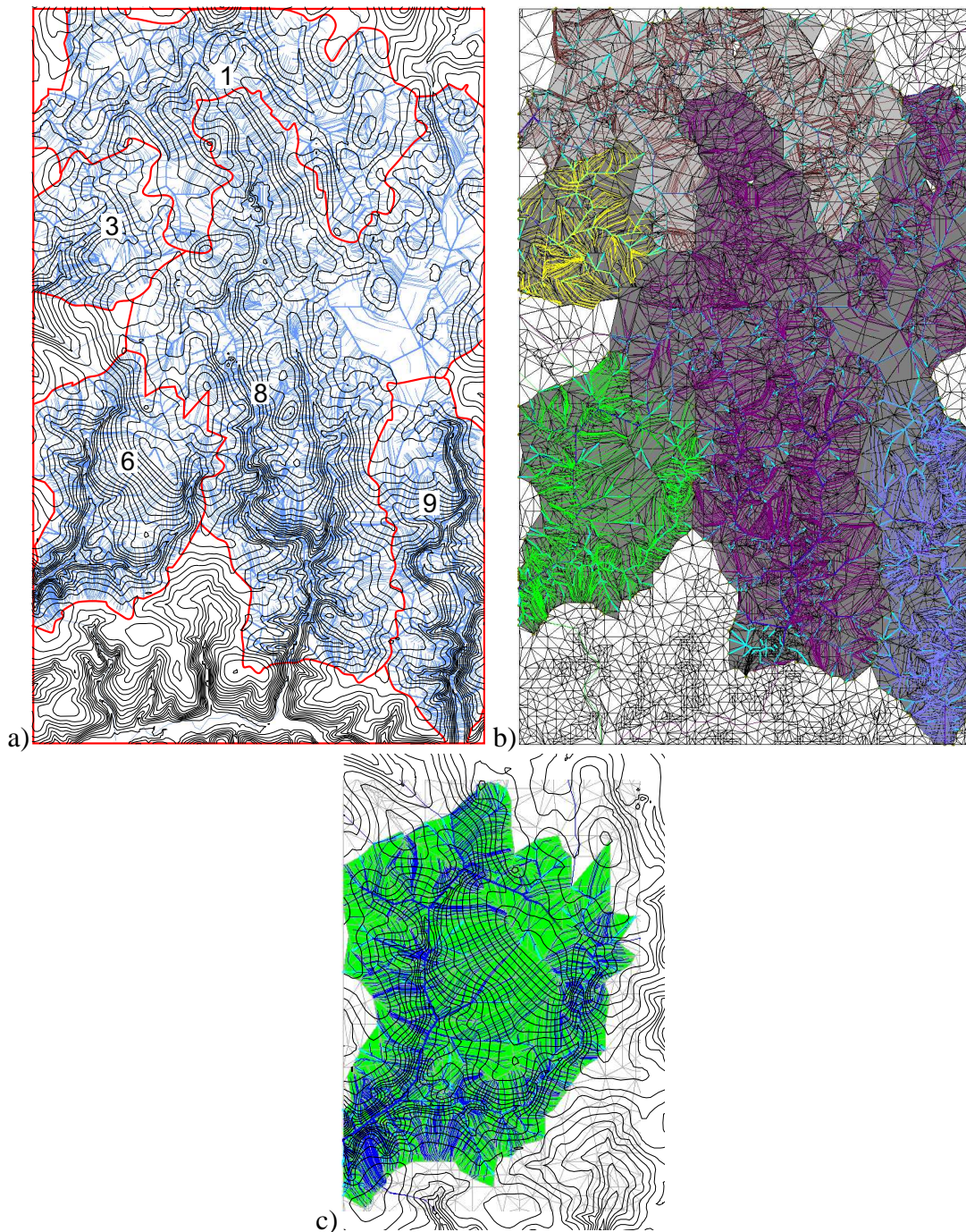
**Table 3:** Attributes of DEM2’s basins (figure 11).

ArcView			TINMOD		
Basin	Area (km <sup>2</sup> )	Flowlength (km)	Basin	Area (km <sup>2</sup> )	Flowlength (km)
1	16.98	10.69	1	16.76	10.99
2	16.02	18.15	–	–	–
3	5.34	4.23	3	5.37	4.73
4	18.65	14.46	–	–	–
5	7.37	14.57	–	–	–
6	11.29	6.07	6	11.43	6.23
7	40.81	8.63	–	–	–
–	–	–	8	26.57	17.15
–	–	–	9	9.06	8.97





**Figure 12:** Basin watersheds and drainage directions compared with contour maps, for DEM1. (a) Drainage flowpaths (blue) for all the basins, basin watersheds (red) traced from *b*, overlain by contours. (b) Drainage flowpaths for each basin, using the same colouring scheme as figure 10. (c) Detail of basin 4, showing that the basin watershed coincides with peaks and ridges in the landscape and that water drainage into valleys and away from ridges. The apparent ‘stray’ blue drainage path leaving the basin, is probably due to a node being on the watershed, having more than one outlet).



**Figure 13:** Basin watersheds and drainage directions compared with contour maps, for DEM2. (a) Drainage flowpaths (blue) for all the basins, basin watersheds (red) traced from *b*, overlain by contours. (b) Drainage flowpaths for each basin, using the same colouring scheme as figure 11. (c) Detail of basin 6 which, as did figure 12 shows, shows that the basin watershed coincides with peaks and ridges in the landscape and that water drains into valleys and away from ridges.

Figure 14 shows that the constrained TIN has performed better at delineating DEM1's basins (note that no basins have been dissolved). Close to fully connected networks are represented and the whole of DEM1's main basin has been delineated. The unconstrained TIN has fragmented this basin because of breaks in the channel network, which are often 'bridged' by small sections of overland flow. Section 2.2.3 discusses why this should be avoided for hydrological models. Note that in most cases, acceptable basins can be delineated by dissolving smaller basins (by filling their pit); this is dealt with in the next section.

#### 4.4 Basin Dissolution and Basin Outlet Choice

Issues of basin outlet selection and basin dissolution were discussed in section 3.6. TINMOD can automatically dissolve basins below a threshold size. Section 3.6 explained that this may not always give the best result because it is sensitive to the order in which the basins are dissolved. The most sure way is to interpret a contour map, and select basins to dissolve manually.

In the previous section, the unconstrained TIN could not fully delineate DEM1's largest basin. Figure 16 shows that the small basins which were in its place could be dissolved by hand, to produce an acceptable basin.

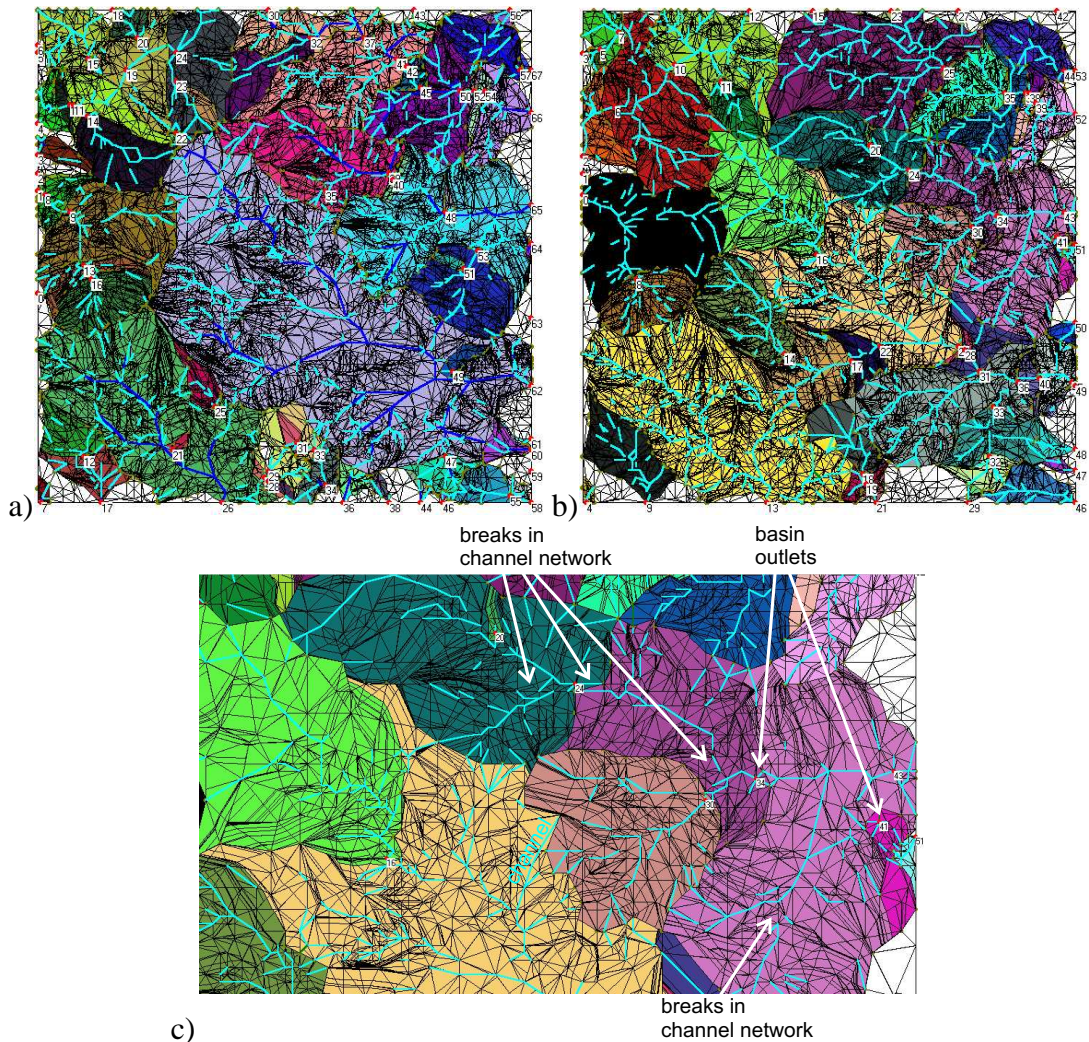
In section 4.2.2 and figure 13a, it was shown that TINMOD's channel extraction algorithm failed to identify the largest channel in the TIN of DEM2, due to its sensitivity to scale. As a result, even though the TIN in figure 13a is constrained (dark blue edges), this channel is not. Figure 17 shows that manual basin dissolution can successfully delineate the full basin, but because there are no constrained channels, the channel network is broken.

#### 4.5 Sensitivity to Spatial Resolution

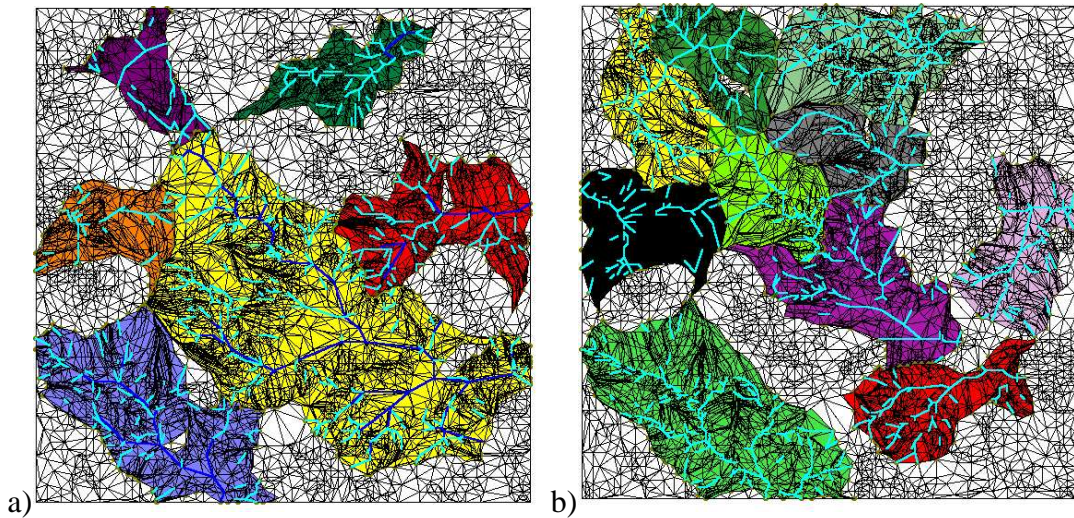
In the tests so far, the 'hierarchy(40)' method has been used for the second stage of constrained TIN-building. This series of tests assesses the output differences of using both the 'hierarchy method' and the 'VIP' method', at different tolerances (see section 4.1).

All the methods produce similar results, in particular, basins 2, 3 and 6, with the notable exception of a miscalculation of the area of basin 4, for 'hier5' and the VIP methods (this apparent error with TINMOD is not discussed here). It seems that the other errors are more to do with basin delineation errors (see next paragraph), rather than the sensitivity of the area and flowlength algorithms to sizes of 'facet areas'. Since facet can be subdivided, the overestimation of areas which would be expected in a simple raster model (like the one implemented in ArcView), is avoided to a certain extent.

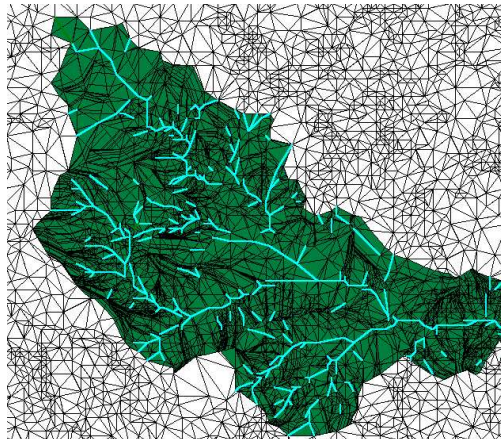
Delineating basins using the 'Hierarchy5' method and the VIP methods was much more difficult because there were more spurious pits (as seen in table 4) and basins did not easily dissolve into each other. This was mostly a problem for unconstrained basins such as basin 6. Figure 19a, illustrates that the TIN is so fine that it has reached the resolution of the raster in many places. The regular distribution of nodes, means



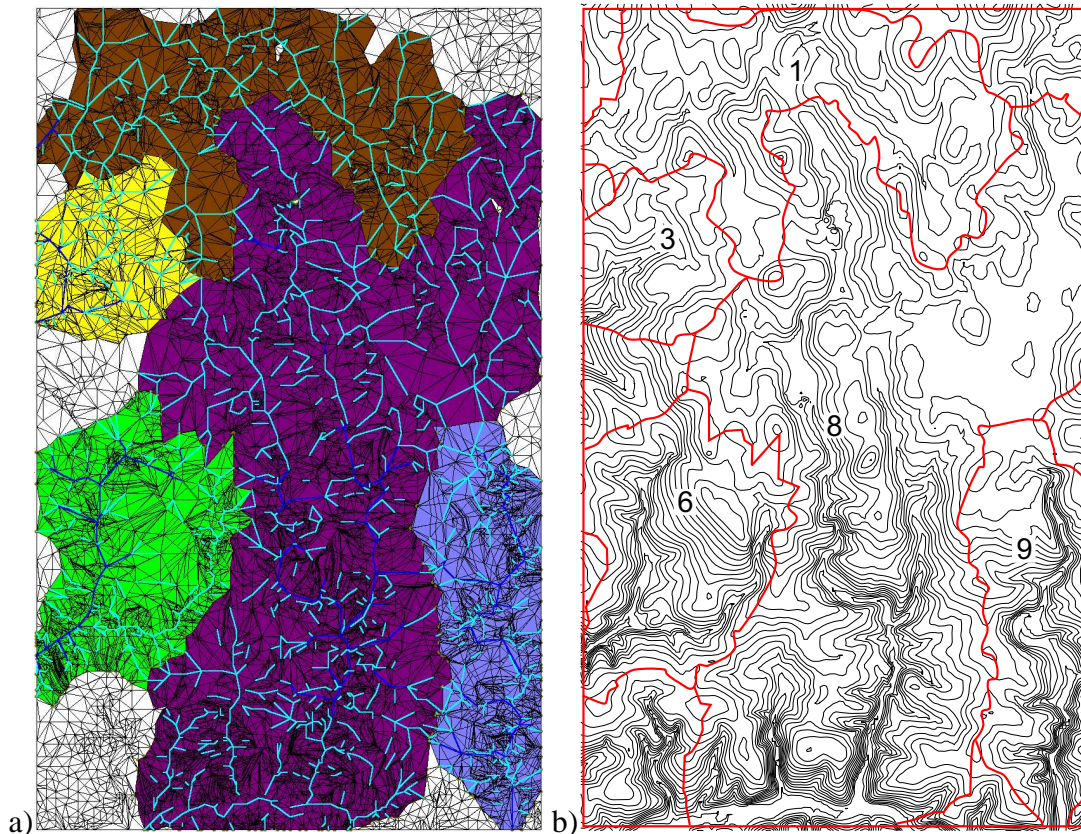
**Figure 14:** Comparison of a constrained with an unconstrained TIN for DEM1. Note that *no* basins has been dissolved. Constrained channels are in dark blue; unconstrained channel in light blue. (a) Constrained TIN, with the basins of all 68 pits delineated. (b) Unconstrained TIN, with basins of all 54 pits delineated. Note that the significant difference between the constrained and unconstrained TIN is that the former has close to a *fully connected network* for the major basins and it has managed to delineated almost the *whole* of the largest basin (from pit 62). The ‘holes’ in this and other basin are caused by pits which should not be there, effectively forming a sub basin. Note that it is possible for basins to overlap, if a note on a watershed has two outlets into different basins (once small basins have been dissolved, this is rare). (c) Detail of b, showing the unconstrained and broken river network.



**Figure 15:** Basins greater than  $2\text{km}^2$  for a constrained and an unconstrained TIN for DEM1. (a) Constrained TIN (figure 14a). (b) Unconstrained TIN (figure 14b). With the exception of the basin at the lower left of the TIN, the constrained TIN has better delineated DEM1's basins.



**Figure 16:** Manually dissolving basins. Acceptable basins can be delineated by careful choice of small basins to dissolve. This example is the result of dissolving the small basins in figure 14b, to produce a large one, with an area of  $12.18\text{km}^2$  and a flowlength of  $8.02\text{km}$ . The basins (labelled in figure 14b) dissolved were 16, 27, 14, 17, 22, 28, 31, 36, 40 and 45. Note that the equivalent basin delineated by the constrained TIN produces a better result, because the river network is fully connected.



**Figure 17:** TINMOD basin outputs for DEM2, with basin 8 at its full size. (a) TINMOD basin outputs for DEM2, with basin 8 at its full size. It is now  $36.16\text{km}^2$  and has a maximum flowlength of  $21.40\text{km}^2$ . (b) Outlines of the basins overlain onto the contour map. Note the full extent of basin 8, compared to figure 11. The main channel at the outlet of basin 8 is too wide to be identified as a channel by the overlying raster-processing algorithm of TINMOD for extracting channel. Thus, this part of the TIN is unconstrained, and a fully connected basin network does not exist. It contained a series of small basins and spurious pits, which had to be dissolved by hand. Note that since the main channel's basin is incomplete, this basin represents only a partial basin. It would probably be better to delineate the basin from the main tributary, similar to the basin in figure 11, but a bit further south.

**Table 4:** Comparison of different TIN point selection algorithms at different tolerances. (a) Characteristics of the resulting TIN. (b) Areas of the basin delineated from these TINs (see figure 18 for basin key and graphs). (c) Maximum flowlengths of the same.

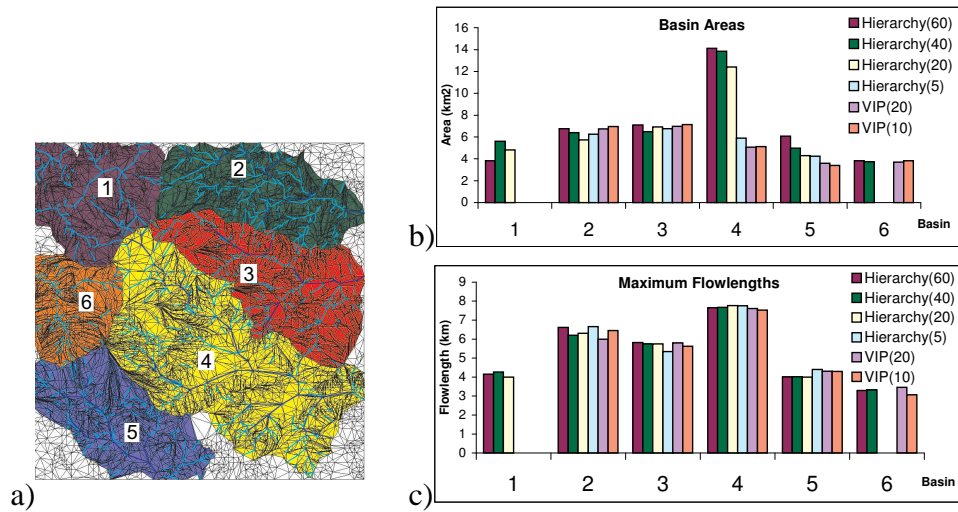
<b>Algorithm (tolerance)</b>		<b>Points used</b>	<b>Mean error (cm)</b>		<b>Initial number of pits</b>	
a)	Hierarchy(60)	13%	18.87		44	
	Hierarchy(40)	20%	13.26		65	
	Hierarchy(20)	37%	7.36		87	
	Hierarchy(5)	70%	3.29		112	
	VIP(20)	47%	13.29		116	
	VIP(10)	62%	5.82		125	

<b>Areas of Basins</b>							
	<b>Basin 1</b>	<b>Basin 2</b>	<b>Basin 3</b>	<b>Basin 4</b>	<b>Basin 5</b>	<b>Basin 6</b>	
b)	Hierarchy(60)	3.83	6.77	7.10	14.12	6.08	3.83
	Hierarchy(40)	5.61	6.40	6.49	13.85	4.98	3.74
	Hierarchy(20)	4.82	5.74	6.92	12.41	4.29	
	Hierarchy(5)		6.26	6.77	5.90	4.23	
	VIP(20)		6.75	6.98	5.09	3.59	3.71
	VIP(10)		6.97	7.14	5.14	3.39	3.84

<b>Maximum Flowlength of Basins</b>							
	<b>Basin 1</b>	<b>Basin 2</b>	<b>Basin 3</b>	<b>Basin 4</b>	<b>Basin 5</b>	<b>Basin 6</b>	
c)	Hierarchy(60)	3.83	6.77	7.10	14.12	6.08	3.83
	Hierarchy(40)	5.61	6.40	6.49	13.85	4.98	3.74
	Hierarchy(20)	4.82	5.74	6.92	12.41	4.29	
	Hierarchy(5)		6.26	6.77	5.90	4.23	
	VIP(20)		6.75	6.98	5.09	3.59	3.71
	VIP(10)		6.97	7.14	5.14	3.39	3.84



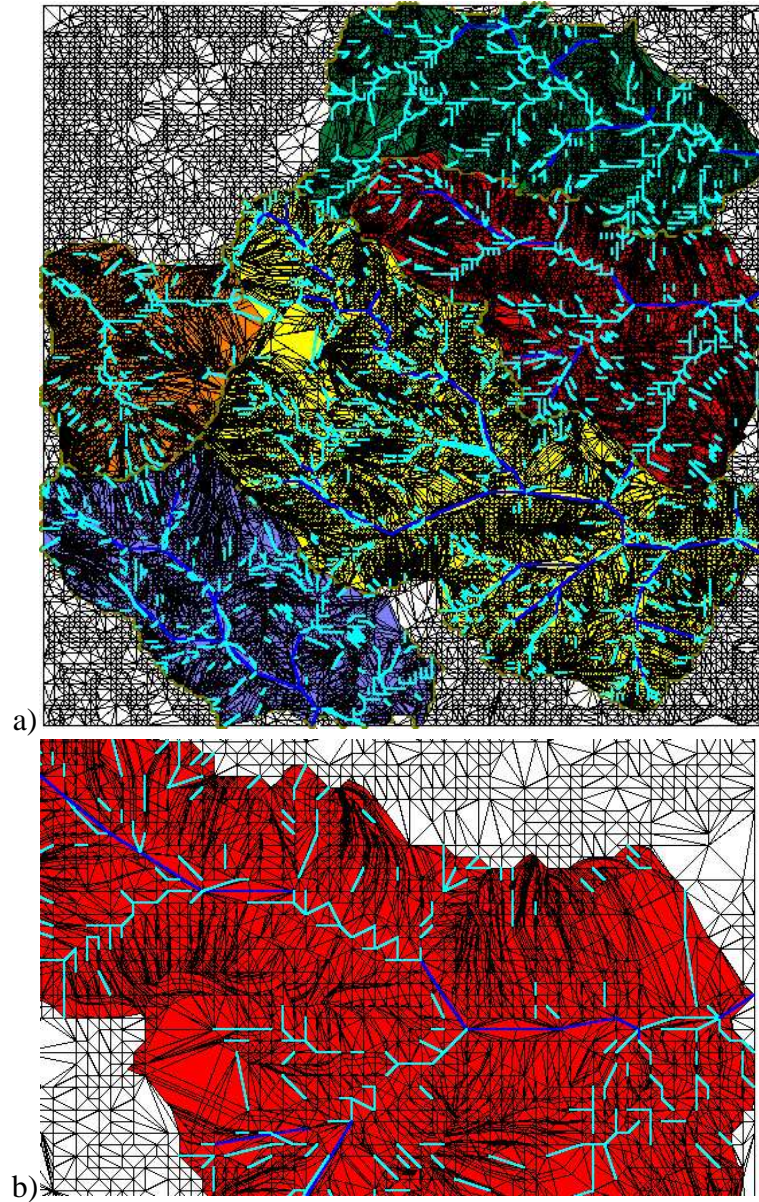
**Figure 18:** Graphs of area and maximum flowlength for six basins of DEM1, using two triangulation algorithms at different tolerances. (a) Key for basin numbers. (b) Areas of basins 1–6, computed by TINMOD for constrained TINs triangulated by using the ‘hierarchy method’ and the ‘VIP method’ at the tolerances (indicated in brackets – but note that they are not comparable between method). All the methods give similar areas, with the exception of the hierarchy(5) method and the VIP methods, which miscalculate the area of basin 4. (c) As b for maximum flowlengths. All the methods give similar results.

that there is no preferred orientated for Delaunay triangulation. Because the edges are so short and their configuration is arbitrary, it is less likely that a fully connected unconstrained network will be represented. As seen in section 4.1, this problem would tend to apply to TINs produce using the VIP method and TINs with a very low mean error (<5cm).

Figure 19b shows detail of a TIN at the resolution of the raster. Note for areas of the TIN at this resolution, edge directions are restricted to 45° increments, leading to the original raster’s geometrical structure being imposed on simulated channel networks – one of the problems which the use of TINs aimed to reduce. However, overland drainage directions do not suffer this restriction.

So, it is interesting to note that the use of TINs with a low mean errors at best, has limited benefit, and at worst has a *detrimental* effect on TINMOD With rasters, this would not normally be the case. Benefits are limited because the limited benefit is because the most *significant* points in the landscape *are* represented even higher mean errors and basins are delineated and modelled at a *higher* spatial resolution to the TIN facets. The detrimental effect occurs at very low mean errors (<5cm), where parts of the TIN reach the resolution of the original raster. Because channels are represented as edges, the geometrical artifacts are introduced into channel networks.





**Figure 19:** Problems with TINs at a high spatial resolution. (a) This TIN is so fine, that in many places, it is at the spatial resolution of the underlying raster, and the regular raster geometry can be seen. (b) The fact that edges follow  $45^\circ$  drainage direction increments and that channels are represented as edges, channel geometries have this restriction imposed on them a similar problem to that of rasters.

## 5 Conclusions

Using TINMOD, this study has shown that hydrological modelling using TINs (as an irregular data structure) and an object-orientated approach is both technically feasible and may offer advantages to raster based models.

TINMOD produces comparable results to a simple raster-based model in ArcView. Its watersheds and drainage directions agree with a contour map of the same DEM; watersheds connect peaks and match ridges and the basin drainage directions are perpendicular to the contours. The importance of constrained TINs for representing fully connected river networks is illustrated and it is shown that they do, in general, allow the more effective delineation of basins, though the channel extraction algorithm seems to be scale-dependent. The ability to dissolve (merge) basins is essential, and a semi-automatic approach to this often gives the best results. Finally, the sensitivity of the model to different spatial resolutions is tested, by running the model on TINs with different mean errors. It is found that TINs with a lower mean error may have only limited benefits, and TINs whose mean error is very low (<5cm), actually have a detrimental effect on the basin delineation and TINMOD's simulated channels, producing artifacts of the geometrical structure of the original DEM.

The next stage in development would be to add a flow-velocity based model and try and derive unit hydrographs from TINMOD, with a view to eventually implement a full hydrological model. This study addresses and evaluates some important and interesting issues relating to the representation of terrain for hydrological modelling and the initial results are very promising.

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